EXAM ACHARYA
EDUCATION REDEFINED

GPSC - CIVIL

Structural Analysis

"All of us do not have Equal Talent.
But, all of us have an Equal Opportunity
to Develop our Talents."
A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

INDEX

CHAPTER – 1

STRUCTURE

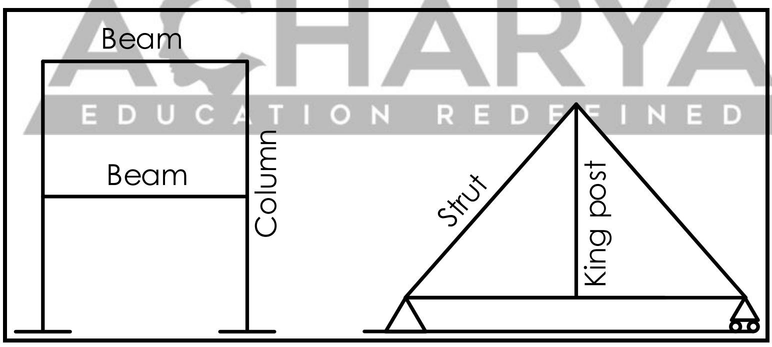
It is the combination for channeling of loads that are resulted from presence and use of building in relation to the ground.

CLASSIFICATION

On the Basis of Geometry

Skeleton Structure

In this structure one dimension is predominant (length of beam or height of column) and other two dimensions are negligible. e.g. building frame and struts.



Surface Structure

In this structure two dimensions are predominant, and one dimension is negligible. e.g. slab, load bearing walls etc.

Solid Structure

In this structure three dimensions are predominant. e.g. massive foundation or machine foundation.

On the Basis of Stiffness

Rigid structure

Structure which don't undergo large deformation. e.g. stone masonry, steel beams and columns.

Flexible Structure

Structure which can undergo large deformation. e.g. suspended cable, bridges, wooden structure etc.

STRUCTURAL ANALYSIS

Analysis is the process of understanding the basic behavior of a structure in terms of its response to the use. During the structural analysis following steps are considered:

1. Identify the loading
2. Determination of support reactions, i.e. external reactions
3. Determination of internal reactive forces, i.e. internal reactions
4. Checking the adequacy of members

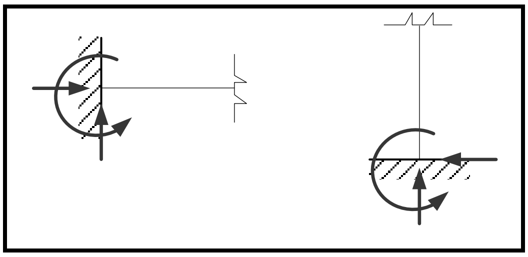
TYPES OF SUPPORT AND ITS REACTIONS

Reaction

Resistance against displacement is known as reaction.

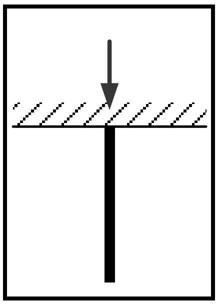
Resistance against rotation is known as moment of resistance.

Fixed support



Reaction = 03 (M, H, V)

Simple support



Reaction = 01 (V)

THEOREM OF EQUILIBRIUM

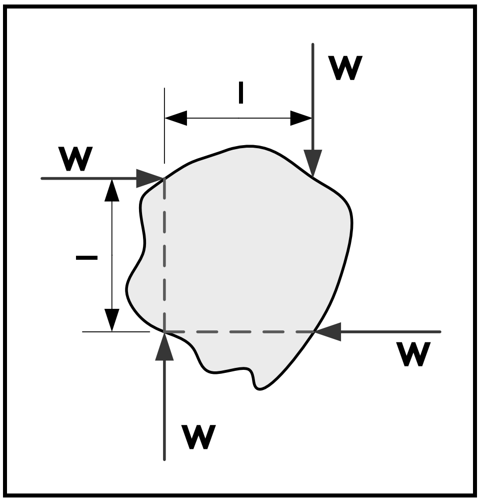
It states that if a body is under equilibrium then three conditions must be satisfied

1. Sum of vertical forces is zero, $\sum V = 0$
2. Sum of horizontal forces is zero, $\sum H = 0$
3. Moment about any point of the body is zero, $\sum M = 0$

> **Note**
> The equilibrium of a body means that, it can't displace from its basic coordinate system in any direction and also it can't rotate about any point such a situation is only possible when net forces in each directions are zero and also net couples on the body are zero.

e.g.:

1. Conclusion for the given body:



Net vertical forces = 0

Net horizontal forces = 0

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Building Material and Construction

Dream is not that which you see while sleeping it is something that does not let you sleep.

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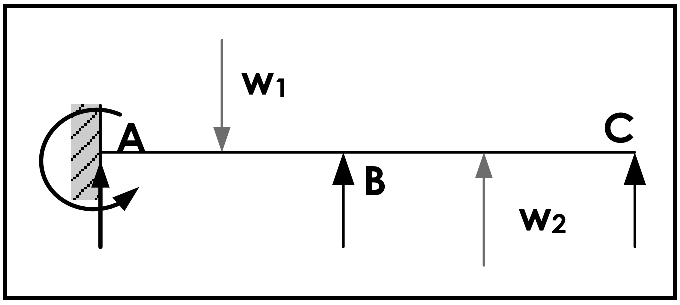
Vertical Loading



No. of reactions = 04

No. of equilibrium condition = 02

The above discussion is valid only for beam structure. In all other structure like frames, truss and arches horizontal reactions are generated on the vertical loading also.

The static indeterminacy can be classified into two categories:

External Static Indeterminacy (D_{se})

All the unknown external reactions (support reactions) in addition to the conditions of equilibrium is known as external static indeterminacy.

From the fig.,

No. of support reactions (r) = 06 (V_A, H_A, M_A, R_B, R_C, and R_D)

No. of equilibrium condition (s) = 03 (for general loading)

$D_{se} = r - s = 06 - 03 = 03$

Internal Static Indeterminacy (D_{si})

No. of internal reactive forces which can't be solved by the theory of equilibrium alone, known as internal static indeterminacy.

In a structure internal reactive forces means the bending moment (B.M.), Shear force (S.F.) and axial forces.

Note

- In case of axially rigid beams, where axial deformation can't occur, the internal static indeterminacy always remains zero, because of all the support reactions are known the B.M. and S.F. can be easily calculated at any section of beam.

From the fig.,

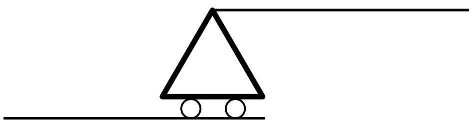
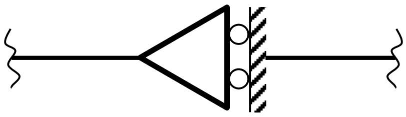
$$\mathbf{D_{si} = 0}$$

$$\mathbf{D_s = D_{se} + D_{si} = 03 + 0 = 03}$$

KINEMATIC INDETERMINACY (D_k)

All the unknowns which are present in the form of movement in a structure are treated as kinematic indeterminacy.

From the figure

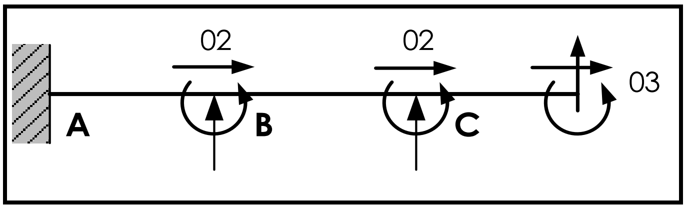
3	Horizontal Roller 	B.M. = 0 A.F. = 0 S.F. ≠ 0 $r_r = 2$
4	Vertical Roller 	B.M. = 0 A.F. ≠ 0 S.F. = 0 $r_r = 2$

Degree of Freedom

No. of independent coordinates where movement is possible is known as degree of freedom.

e.g.:

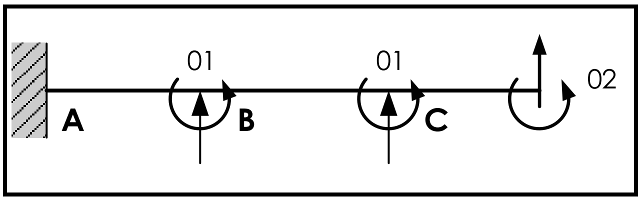
General Loading and Extensible Member



DOF = 0 + 02 + 02 + 03 = 07

> **Note**
> ➢ If members are inextensible then the horizontal displacement is not possible at any point.

GENERAL LOADING AND INEXTENSIBLE MEMBER

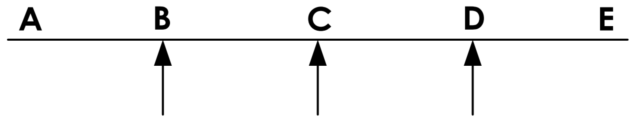


DOF = 0 + 01 + 01 + 02 = 04

$$D_{si} = 0 - (n - 1) = -(02 - 1) = -01$$

$$D_s = D_{se} + D_{si} = 05 - 01 = 04$$

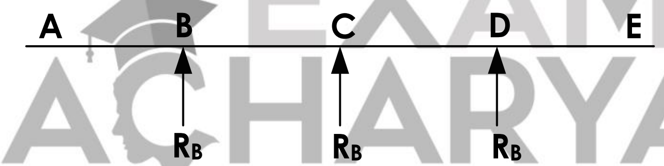
b) For vertical loading and axially rigid member,



Solution

No. of unknown reaction = 03

No. of equilibrium condition = 02



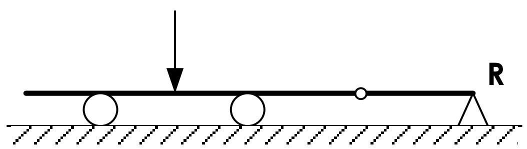
$$D_{se} = 03 - 02 = 01$$

$$D_{si} = 0$$

$$D_s = D_{se} + D_{si} = 01 - 0 = 01$$

Note:
- For vertical loading only horizontal deformations are never considered, even in the case of extensible member because in the absence of horizontal force, horizontal deformation can't occur.

Q2. The static indeterminacy of the two span continuous beam with an internal hinge shown is,



New Batches are going to start.....

Contact:

7622050066

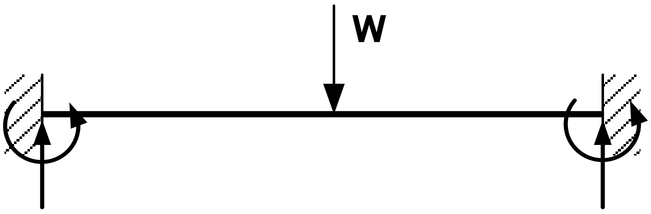
Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
Total full length test	: 13
Mock test	: 16
Total test	: 80

Q4. A beam fixed at the ends and subjected to lateral loads only statically indeterminate and the degree of indeterminacy is,

a. 1 b. 2 c. 3 d. 4

Solution

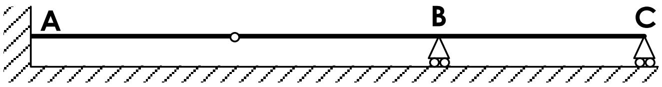


$$\mathbf{D_{se} = r - s = 4 - 2 = 2}$$

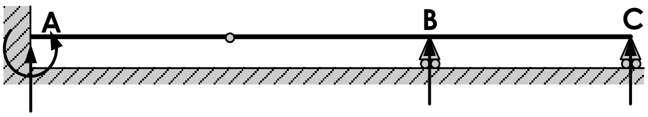
Answer

b. 2

Q5. The degree of static indeterminacy of the beam given below, for vertical loading is ______________.



Solution:



Here,

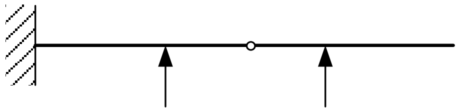
No. of reaction = 4

No. of equilibrium equation req. = 2

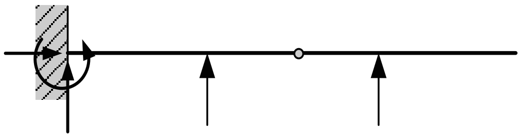
No. of member connected with internal hinge = 2

$$D_{se} = r - s = 4 - 2 = 2$$

Q7. The degree of indeterminacy of the structure as shown in the fig. for general loading is ____________.



Solution



Here,

No. of reaction = 5

No. of equilibrium equation req. = 3

No. of member connected with internal hinge = 2

$$D_{se} = r - s = 5 - 3 = 2$$

$$D_{si} = 0 - (n - 1) = -1$$

$$\mathbf{D_s = 2 - 1 = 1}$$

INDETERMINACY OF FRAMES

Static Indeterminacy

$$\mathbf{D_s = D_{se} + D_{si}}$$

$$\mathbf{D_s = 3C - r_r}$$

(Only applicable for the structure having no internal hinge)

Where,

Dse = External indeterminacy,

Solution

Here,

No. of support reaction = 3 + 3 = 6

No. of equilibrium condition = 3

No. of cuts applied to open the loop = 0

$$\therefore D_{se} = r - s = 6 - 3 = 3$$

$$\therefore D_{si} = 3C = 3 \times 0 = 0$$

$$\therefore \mathbf{D_s = D_{se} + D_{si} = 3 + 0 = 3}$$

Alternative

Here,

No. of cuts applied to obtain the cantilevers = 1

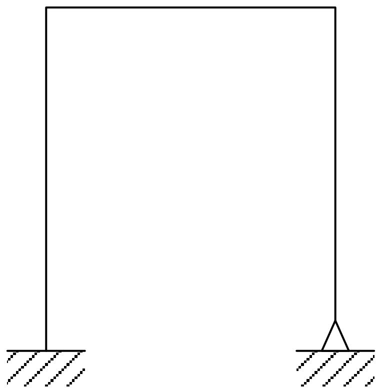
No. of release rection = 0

$$\boldsymbol{D_s = 3C - r_r = (3 \times 1) - 0 = 3}$$

Answer

The static indeterminacy is 3.

b.



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Construction, Planning and Management

"All Birds find shelter during a rain.
But Eagle avoids rain by flying above
the Clouds."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

Solution

Here,

No. of support reaction = 3 + 3 + 2 = 8

No. of equilibrium condition = 3

No. of cuts applied to open the loop = 2

$$\therefore D_{se} = r - s = 8 - 3 = 5$$

$$\therefore D_{si} = 3C = 3 \times 2 = 6$$

$$\therefore D_s = D_{se} + D_{si} = 5 + 6 = 11$$

Alternative

Here,

No. of cuts applied to obtain the cantilevers = 1

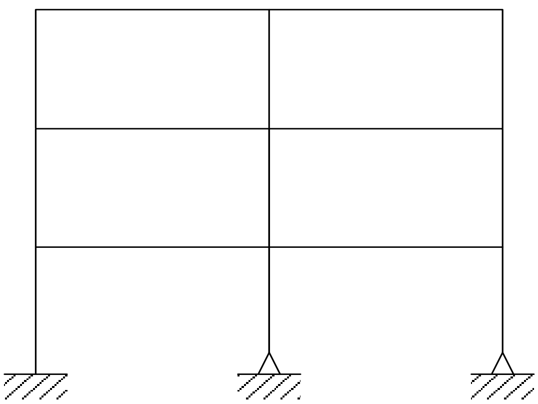
No. of release rection = 1

$$\mathbf{D_s = 3C - r_r = (3 \times 4) - 1 = 11}$$

Answer

The static indeterminacy is 11.

d.



Solution

Here,

No. of support reaction = 3 + 2 + 3 + 1 = 9

No. of equilibrium condition = 3

No. of cuts applied to open the loop = 6

$$\therefore D_{se} = r - s = 9 - 3 = 6$$

$$\therefore D_{si} = 3C = 3 \times 6 = 18$$

$$\therefore \mathbf{D_s = D_{se} + D_{si} = 6 + 18 = 24}$$

Alternative

Here,

No. of cuts applied to obtain the cantilevers = 9

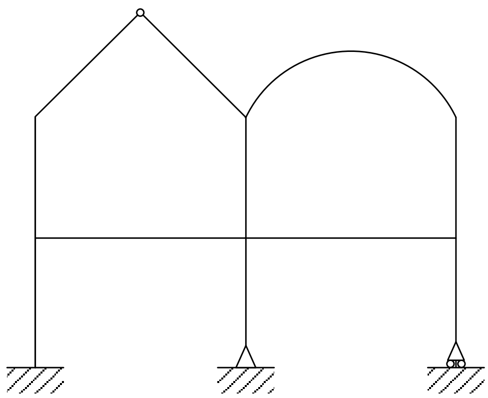
No. of release rection = 1 + 2 = 3

$$\mathbf{D_s = 3C - r_r = (3 \times 9) - 3 = 24}$$

Answer

The static indeterminacy is 24.

f.



$$\mathbf{D_k = 3j - r_e + r_r}$$

Where,

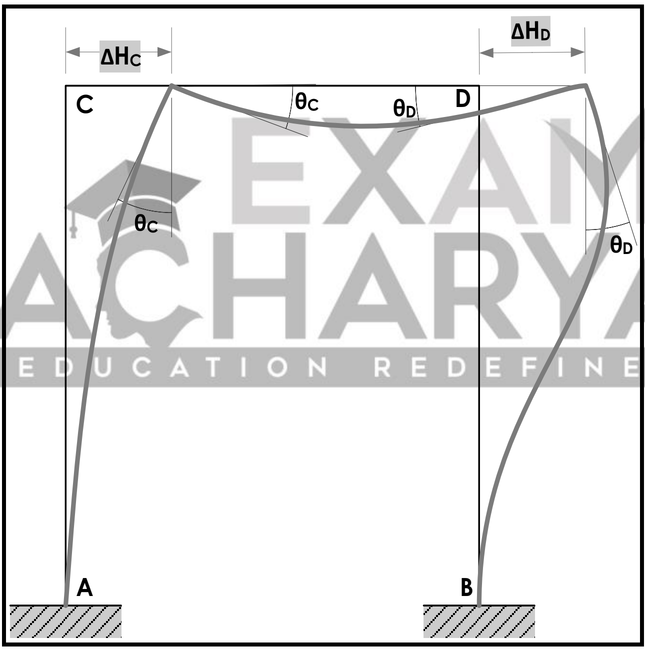
j = No. of joints,

re = No. of support reactions.

r_r = Released reaction

$$\mathbf{D_k = (3 \times 4) - 6 = 6}$$

When Members of Frame are Rigid or inextensible



$D_k = 03$ [θ_C, θ_D, ΔH ($\Delta H_C = \Delta H_D$)]

$$\mathbf{D_k = 3j - r_e - m + r_r}$$

Where,

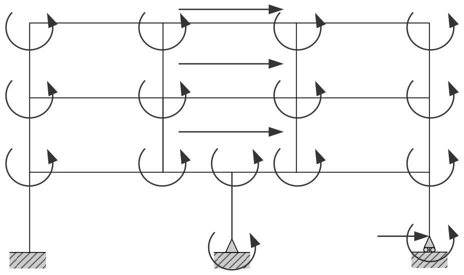
j = No. of joints,

r_e = No. of support reactions.

m = No. of rigid members,

r_r = Released reaction

For inextensible members,

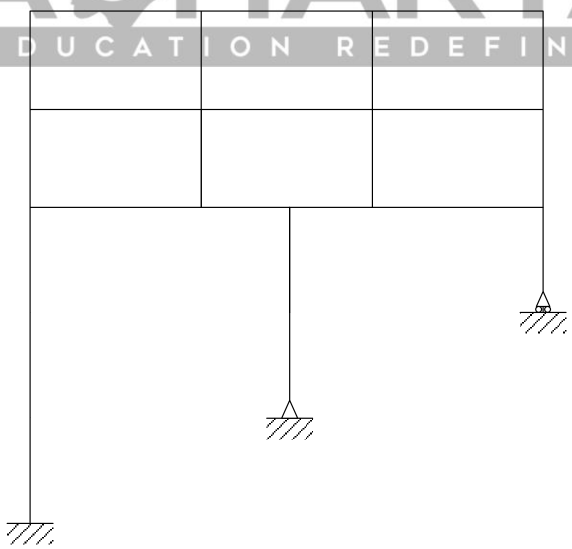


$D_k = 13 + 3 + 3 = 19$

Answer

The kinematic indeterminacy of the structure is 42 and 19 for extensible and inextensible members respectively.

Q2. Find the kinematic indeterminacy of the frame structure in the fig given below. (Assume all the members are inextensible)



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Total weekly test	: 35
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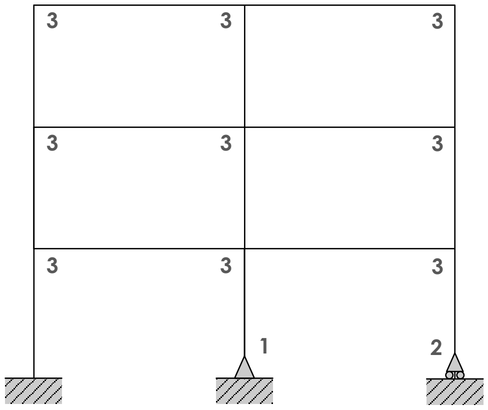
Mock test	: 16
Total test	: 80

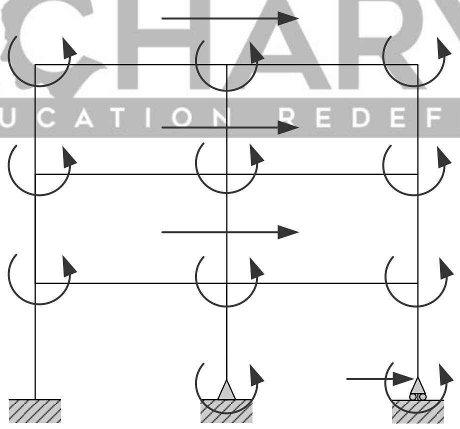
Solution

For extensible members,



$D_k = (9 \times 3) + (1 + 2) = 30$

For inextensible members,



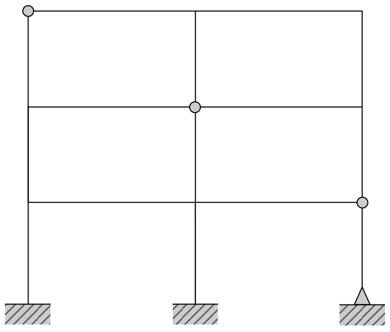
$D_k = 9 + 3 + 3 = 15$

Answer

The kinematic indeterminacy of the structure is 30 and 15 for extensible and inextensible members respectively.

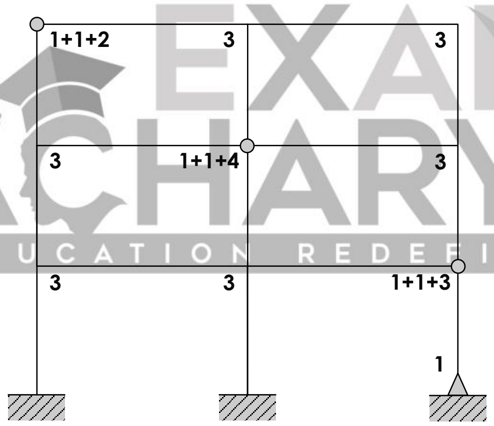
> **Note**
> - The vertical deflection at every point will be zero since none of the columns can contract or expand.
> - The entire frame can deflect in horizontal direction but at same beam level all the joints will deflect in horizontal direction by same amount since none of the beam can contract or expand.

Q5. Find the kinematic indeterminacy of the frame structure in the fig given below. For inextensible and extensible members.



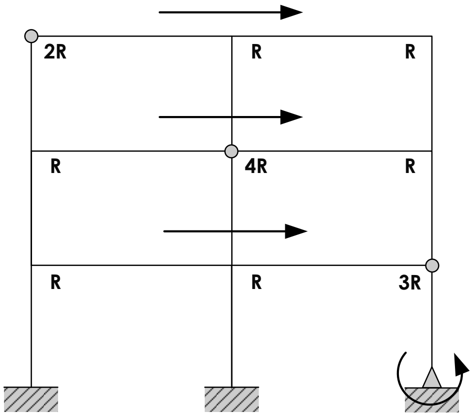
Solution

For extensible members,



$D_k = 34$

For inextensible members,



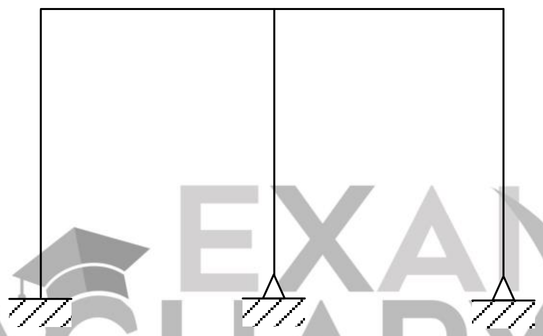
$D_k = 8$

Reduction in kinematic indeterminacy = 14 – 8 = 6

Answer

The reduction in kinematic indeterminacy is 6.

Q7. The degree of static indeterminacy Ns and the degree of kinematic indeterminacy N_k for the plane frame shown below assuming axial deformation to be negligible are given by.



Solution

For static indeterminacy,

$$D_{se} = r - s = (3 + 2 + 2) - 3 = 4$$

$$D_{si} = 3C = 0$$

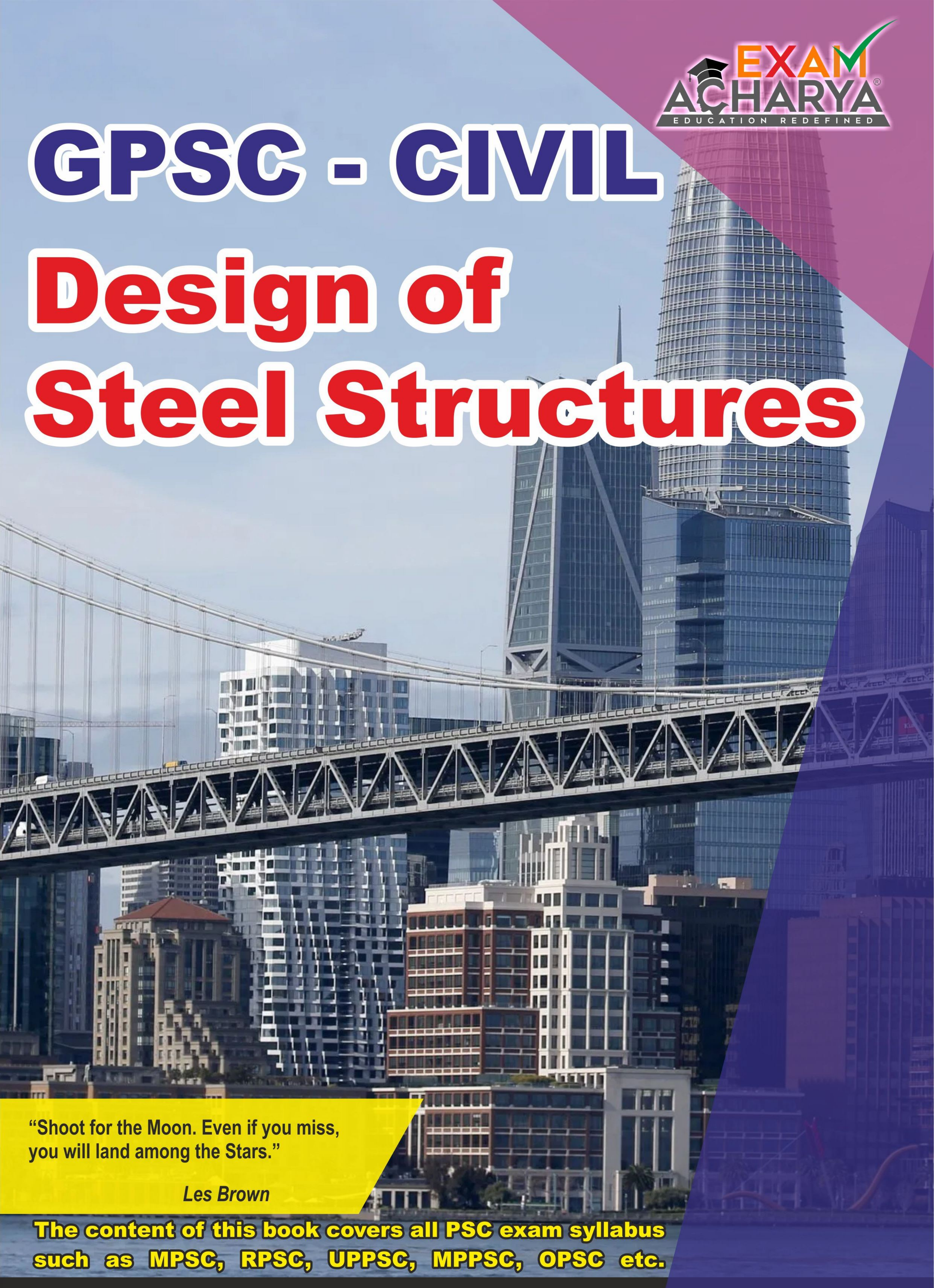
$$\therefore N_s = 4$$

For kinematic indeterminacy.

$$N_k = 3 + 2 + 1 = 6$$

Answer

The value of Ns is 4, and N_k is 6.



GPSC - CIVIL

Design of Steel Structures

"Shoot for the Moon. Even if you miss, you will land among the Stars."

Les Brown

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

Solution

As we know,

$$D_{se} = r - s = (3 + 2 + 2) - 3 = 4$$

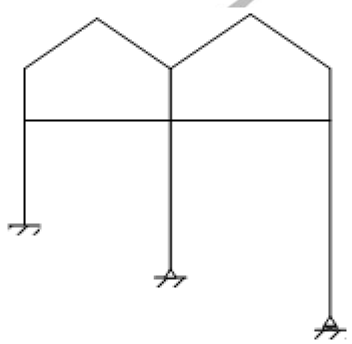
$$D_{si} = 3C = 3 \times 4 = 12$$

$$\mathbf{D_s = 4 + 12 = 16}$$

Answer

The total degree of static indeterminacy of the given structure is 16.

Q11. The static indeterminacy of the structure shown below is _________.



Solution

The static indeterminacy,

$D_s = 3C - r_r$

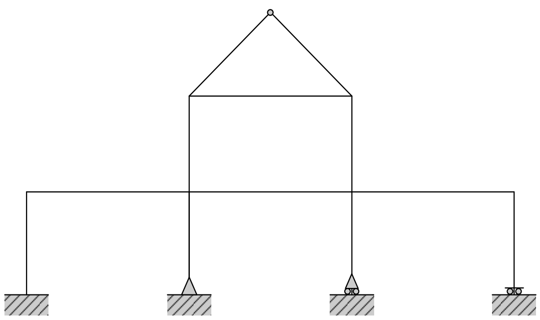
$= (3 \times 4) - (1 + 2)$

$= 9$

Answer

The static indeterminacy is 9.

Q12. The static indeterminacy of the structure shown in the fig. below is ____________

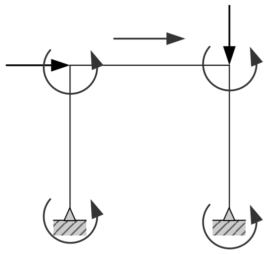


Solution

As we know,

$$D_{se} = r - s = (3 + 2 + 1 + 2) - 3 = 5$$

Solution

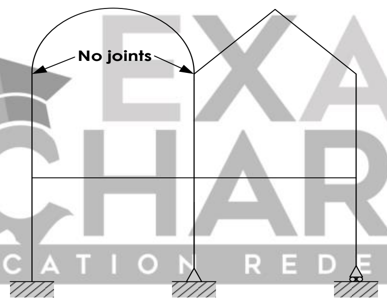


DOF = 5

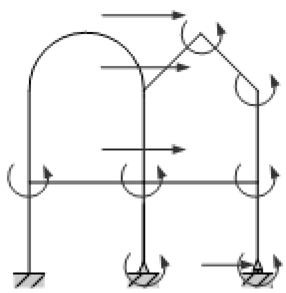
Answer

The degrees of freedom of the structure is 5.

Q15. The degrees of freedom of the structure shown in the fig. below is ____________.



Solution

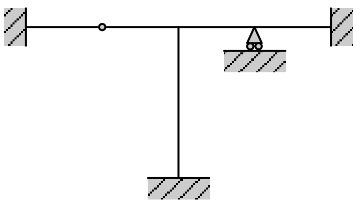


DOF = 11

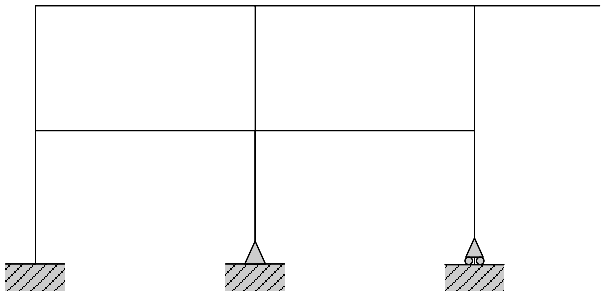
Answer

The degree of freedom is 11.

Q16. The static indeterminacy of the frame given is ____________.



Q18. For the plane frame with an overhang as shown below, assuming negligible axial deformation, the degree of static indeterminacy and degree of kinematic indeterminacy are.



Solution

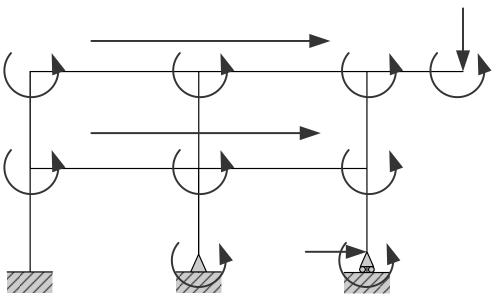
For static indeterminacy,

$$D_{se} = r - s = (3 + 2 + 1) - 3 = 3$$

$$D_{si} = 3C = (3 \times 2) = 6$$

$$\mathbf{D_s = 3 + 6 = 9}$$

For kinematic indeterminacy,



$D_k = 13$

Answer

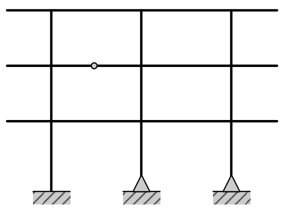
The static and kinematic indeterminacy of the structure is 9 and 13 respectively.

$$D_s = 1 - 1 = 0$$

Answer

The degree of static indeterminacy is 0.

Q21. The degree of static indeterminacy of the plane frame as shown in the fig. is ___________



Solution

$$D_{se} = r - s = (3 + 2 + 2) - 3 = 4$$

$$D_{si} = 3C - I.H = (3 \times 4) - (2 - 1) = 11$$

$$D_s = 4 + 11 = 15$$

Answer

The degree of static indeterminacy is 15.

INDETERMINACY OF TRUSS

Static Indeterminacy

External Static Indeterminacy

$$\mathbf{D_{se} = r - s}$$

Where,

r = No. of support reactions

s = Equilibrium condition

Internal Static Indeterminacy

$$\mathbf{D_{si} = m - (2j - 3)}$$

New Batches are going to start.....

Contact:

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Test Series Available..

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Total test : 80

Solution

$$D_{se} = r - s$$

$$= 4 - 3$$

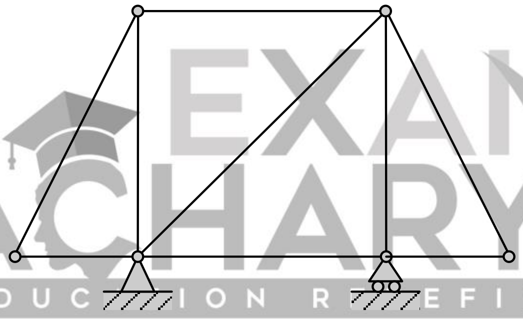
$$= 1$$

Answer

The external static indeterminacy is 1.

Q2. Find the internal static indeterminacy of the trusses in the fig given below.

a.



Solution

$$D_{si} = m - (2j - 3)$$

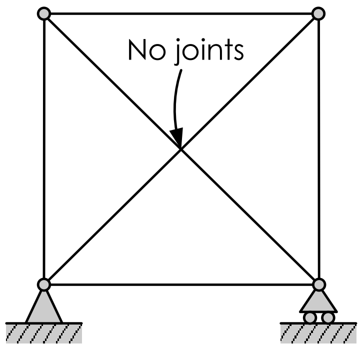
$$= 9 - \{(2 \times 6) - 3\}$$

$$= 0$$

Answer

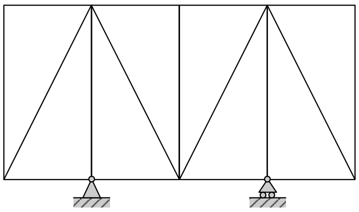
The internal static indeterminacy is 0.

b.

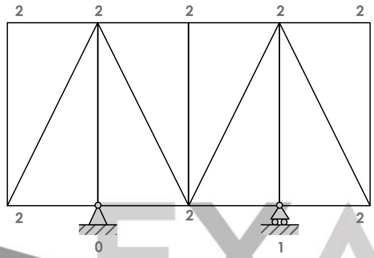


NUMERICAL

Q3. Find the kinematic indeterminacy of the structure



Solution

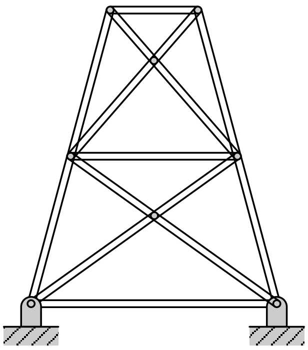


$D_k = 17$

Answer

The kinematic indeterminacy of the truss is 17.

Q4. A Planar truss tower structure is shown in the fig. find the Ds and D_k.



Solution

$$D_{se} = r - s = 4 - 3 = 1$$

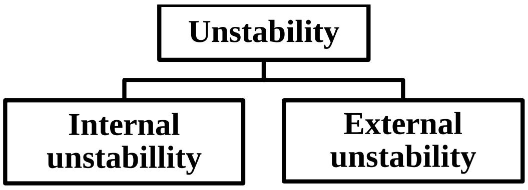
$$D_{si} = m - (2j - 3) = 15 - \{(2 \times 8) - 3\} = 2$$

$$D_s = 1 + 2 = 3$$

CHAPTER – 3

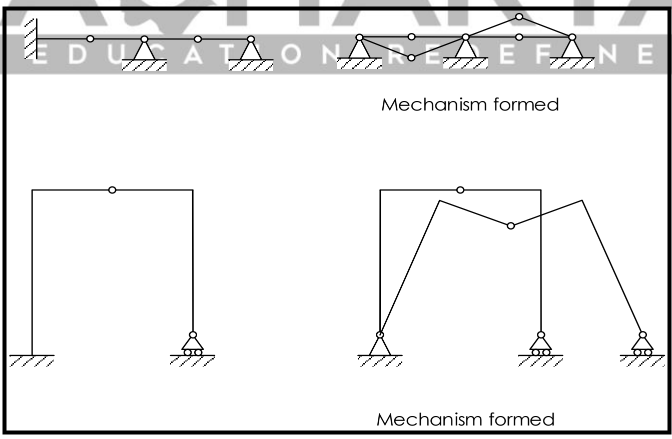
STABILITY OF STRUCTURE

If a structure follows basic three conditions of equilibrium then it is said to be a stable structure and if any of the equilibrium condition is not followed and structure shows the movement, it is known as unstable structure.



INTERNAL UNSTABILITY

If mechanism is formed without loading.



EXTERNAL UNSTABILITY

If structure shows the movement underloading. It is of two types,

Geometric Unstability ($r_e \geq 3$)

1. When all three or more reactions are parallel
2. When all three or more reactions meet at a single point

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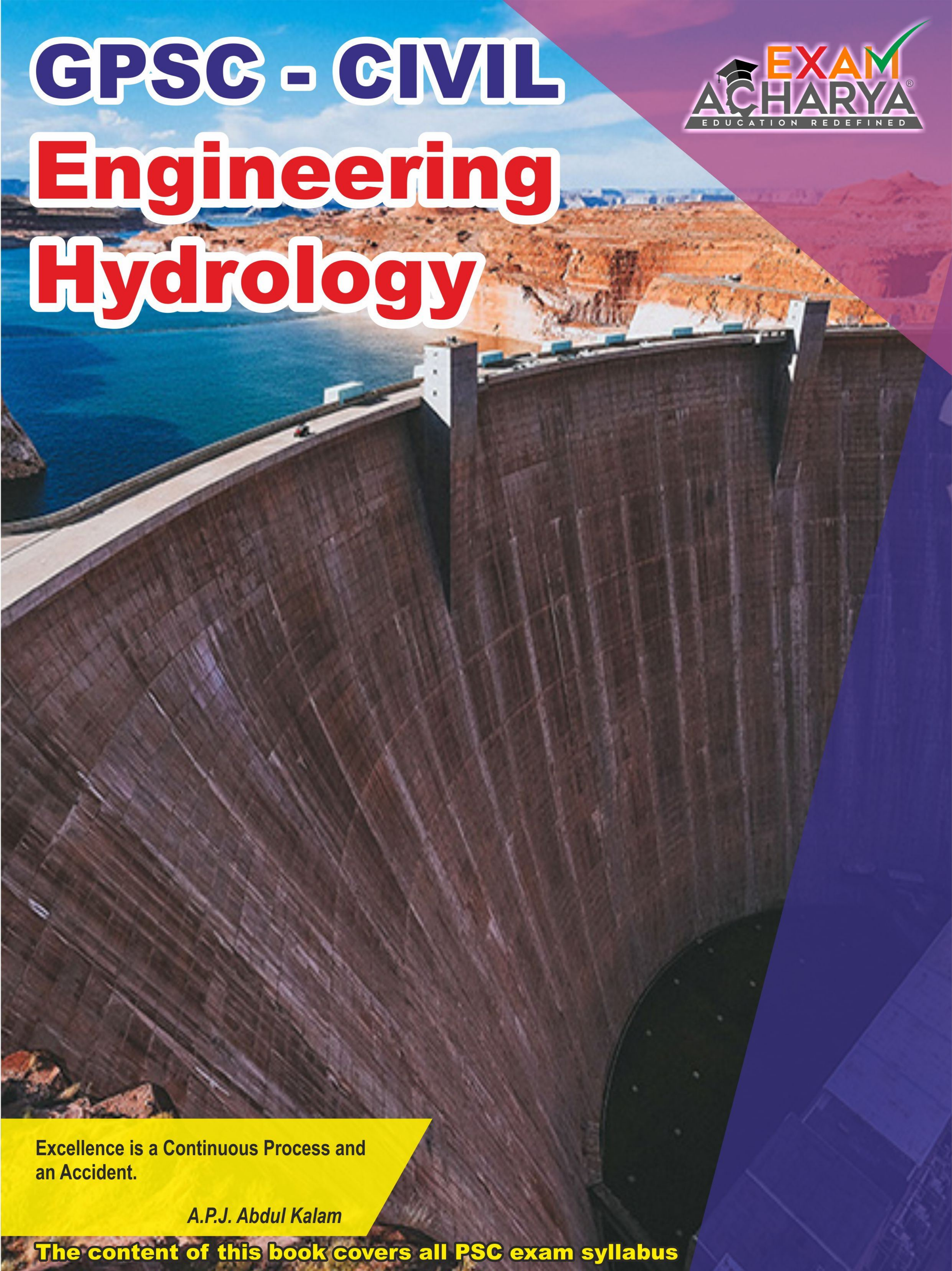
Engineering Hydrology



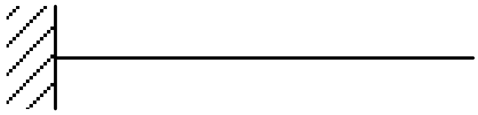
EXAM ACHARYA
EDUCATION REDEFINED

Excellence is a Continuous Process and an Accident.

A.P.J. Abdul Kalam

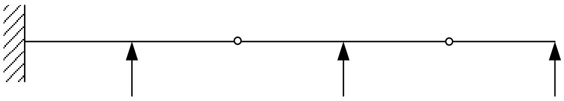
The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

d.



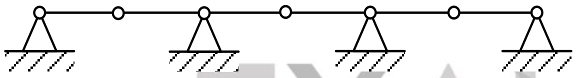
Stable structure

e.



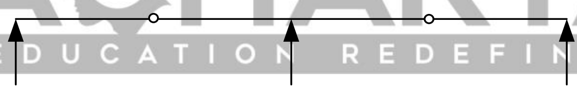
External stable structure and also internal stable structure

f.



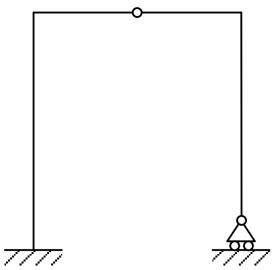
External stable structure but internal unstable structure

g.



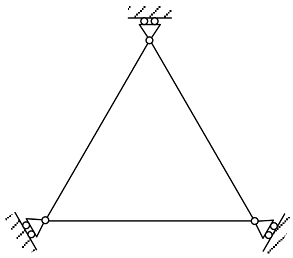
External unstable structure and internal unstable structure.

h.



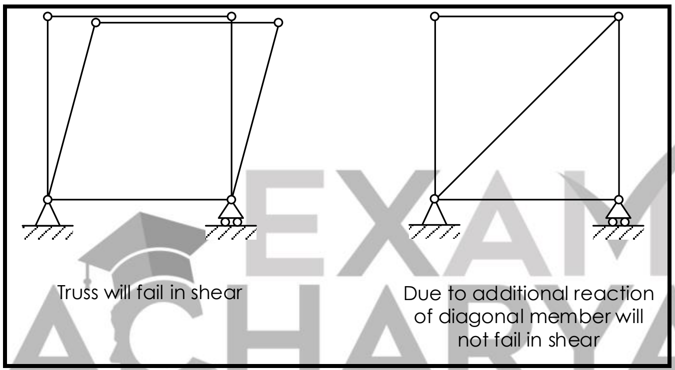
Stable structure

m.



Geometric external unstable.

INTERNAL UNSTABILITY OF TRUSS



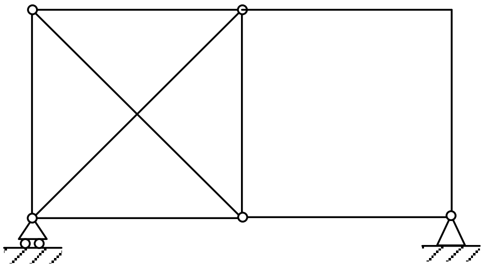
Conclusion

If triangulated form is formed in each panel of truss then it will never fail in shear. If even a single panel is without triangulated form then entire truss will fail in shear due to failure of this panel and the truss will be internally unstable.

NUMERICAL

Q1. For the truss shown in fig. below find whether the truss internal stable or unstable.

a.



Internally unstable.

CHAPTER – 4

ANALYSIS OF TRUSS

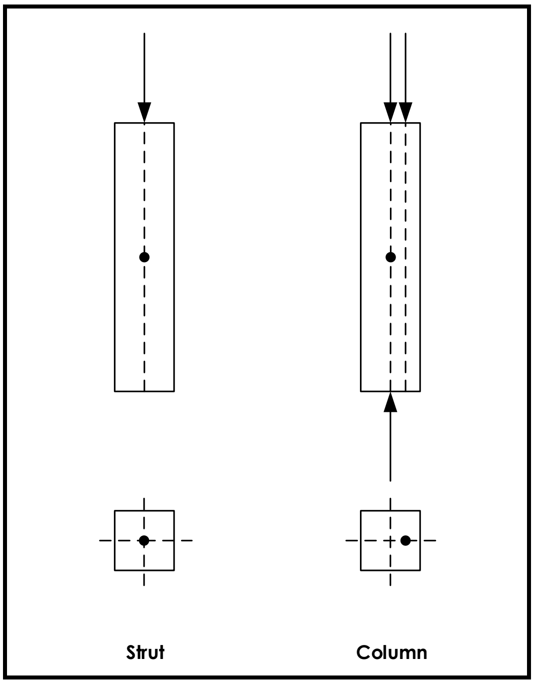
TRUSS

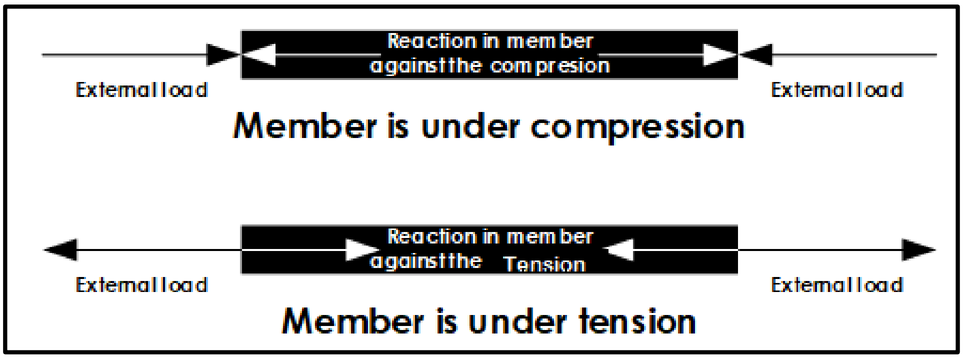
The combination of ties and struts which is designed to carry axial forces only known as truss.

The member, who carries compressive axial force are known as struts and ties are members who carries tensile axial force.

DIFFERENCE BETWEEN STRUT AND COLUMN

Strut is the compression member of truss whereas column is the compression member in a frame. Since trusses are designed to carry axial forces only, hence struts are the members which can carry only axial compression whereas a column can carry axial compression and bending both. Both strut and column can be vertical or inclined.





METHOD OF ANALYSIS

Method of Joints

In this method, equilibrium of each joint is considered separately to solve the truss.

Procedure

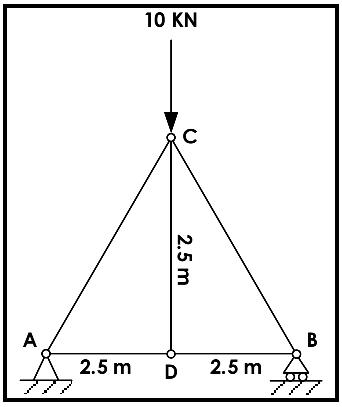
1. Find the support reaction by applying the three conditions of equilibrium of entire truss.
2. Consider the joints separately and obtain the force in members by applying $\sum X = 0, \sum Y = 0$.

Note

- As far as possible don't consider the joints where more than two unknown forces are present (Since we have two conditions of equilibrium at joints).

NUMERICAL

Q1. Find the forces in each member of truss.



New Batches are going to start.....

Contact:

7622050066

Test Series Available..

Total weekly test	**: 35**
Total mid subject test	**: 16**
Total full length test	**: 13**
Mock test	**: 16**
Total test	**: 80**

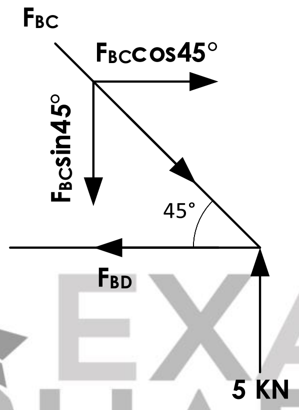
Again,

$$\sum H = 0$$

$$F_{AD} - F_{AC}\cos 45° = 0$$

$$F_{AD} = 5 \text{ KN (T)}$$

Considering equilibrium at joint B,



$$\sum V = 0$$

$$5 - F_{BC}\sin 45° = 0$$

$$F_{BC} = 5\sqrt{2} \text{ KN (C)}$$

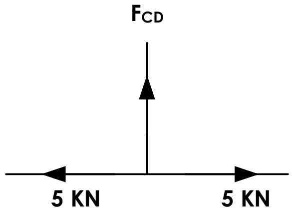
Again,

$$\sum H = 0$$

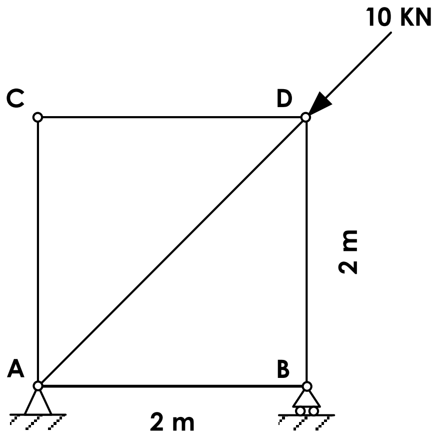
$$F_{BD} - F_{BC}\cos 45° = 0$$

$$F_{BD} = 5 \text{ KN (T)}$$

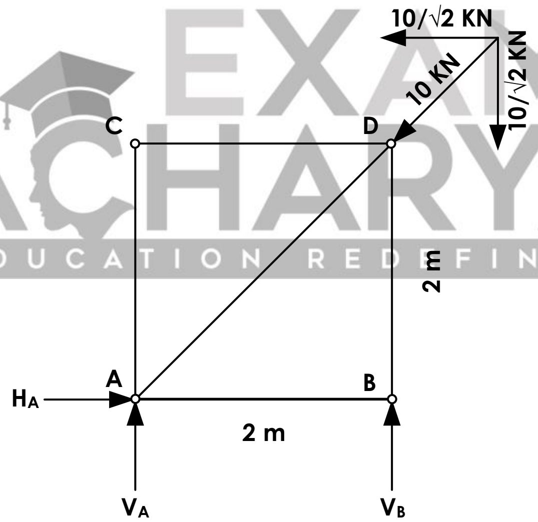
Considering equilibrium at joint D,



Q2. Find force in each member.



Solution



Applying equations of equilibrium,

Taking moment about A,

$$\sum M = 0$$

$$\left(\frac{10}{\sqrt{2}} \times 2\right) - \left(\frac{10}{\sqrt{2}} \times 2\right) - (V_B \times 2) = 0$$

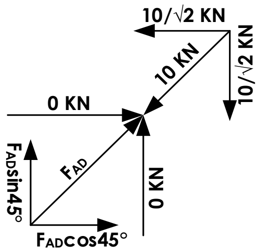
$$V_B = 0$$

$$\sum V = 0$$

$$\sum H = 0$$

$$F_{CD} = 0 \text{ KN}$$

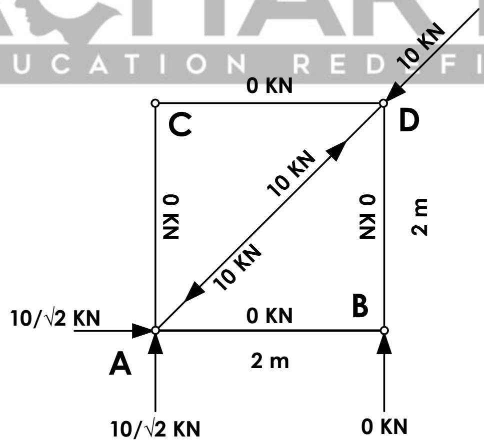
Considering equilibrium condition at joint D,



$$\sum V = 0$$

$$F_{AD} \sin 45° - \frac{10}{\sqrt{2}} = 0$$

$$F_{AD} = 10 \text{ KN (C)}$$



Conclusion

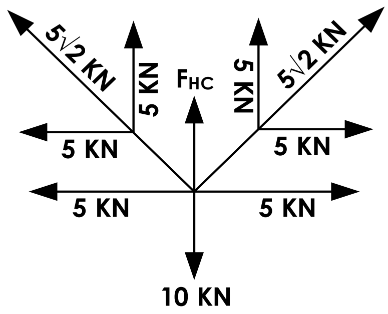
If two members are meeting at a point without carrying any point load, then force in both the member will be zero.

Similarly, we get,

$$F_{IC} = 5\sqrt{2}\ \text{KN (T)}$$

$$F_{IH} = 10\ \text{KN (C)}$$

Now considering equilibrium at joint C,

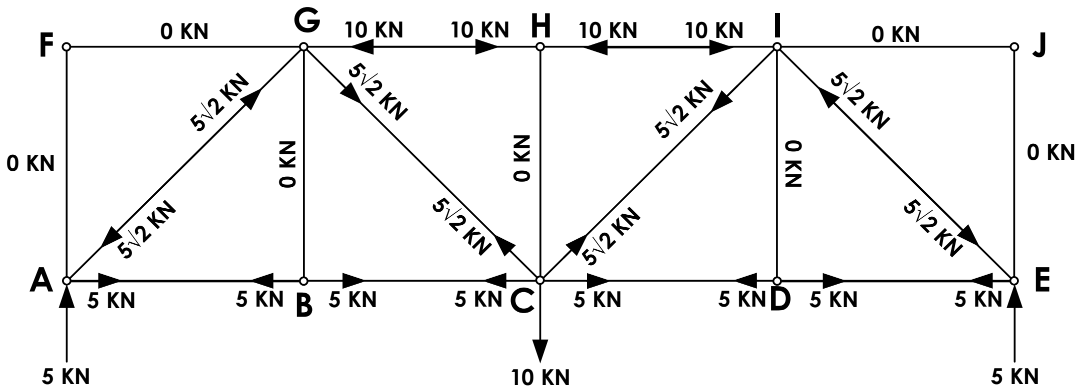


$$\sum V = 0$$

$$F_{HC} + 5 + 5 - 10 = 0$$

$$F_{HC} = 0\ \text{KN}$$

The forces in each members are,



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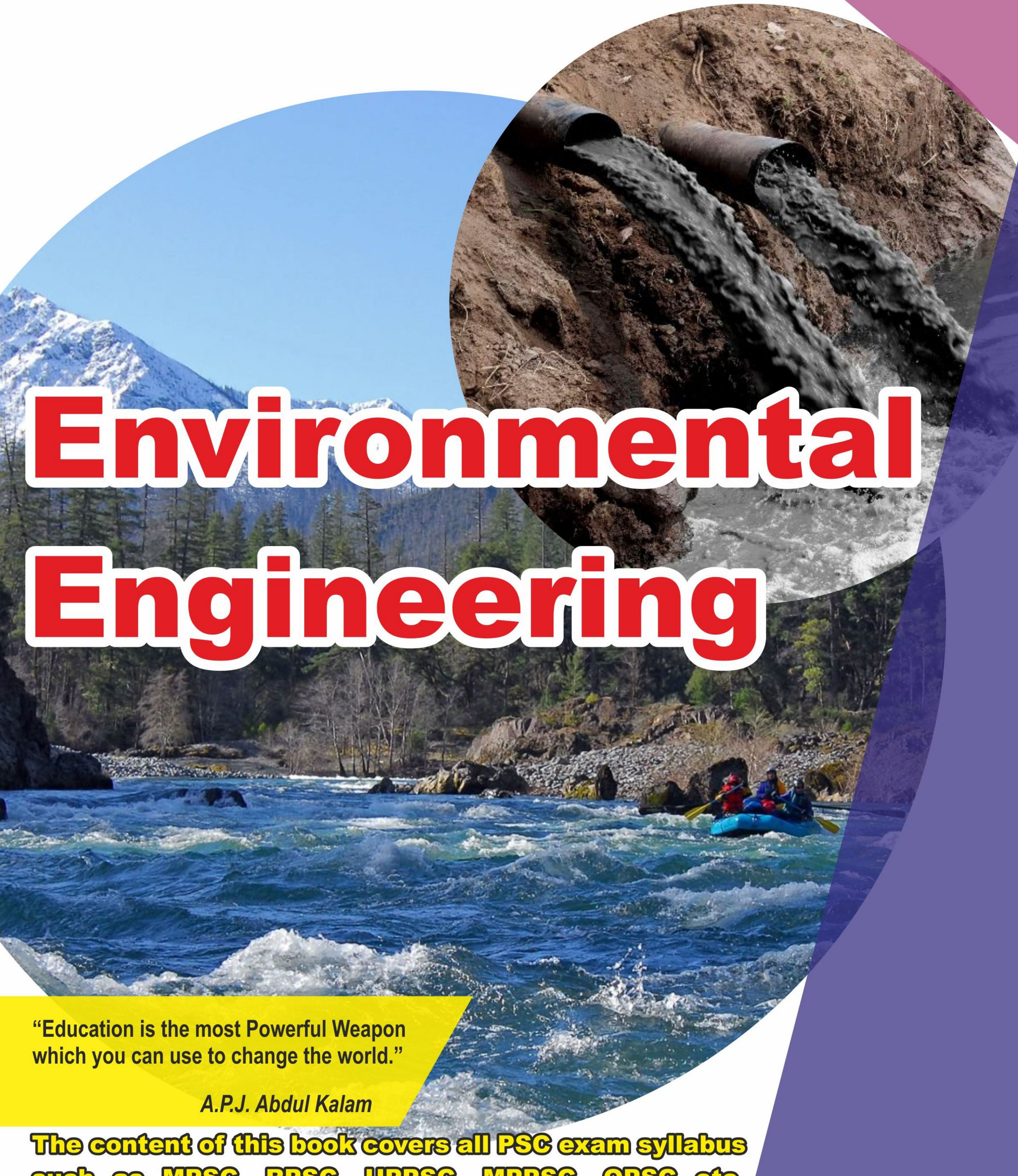


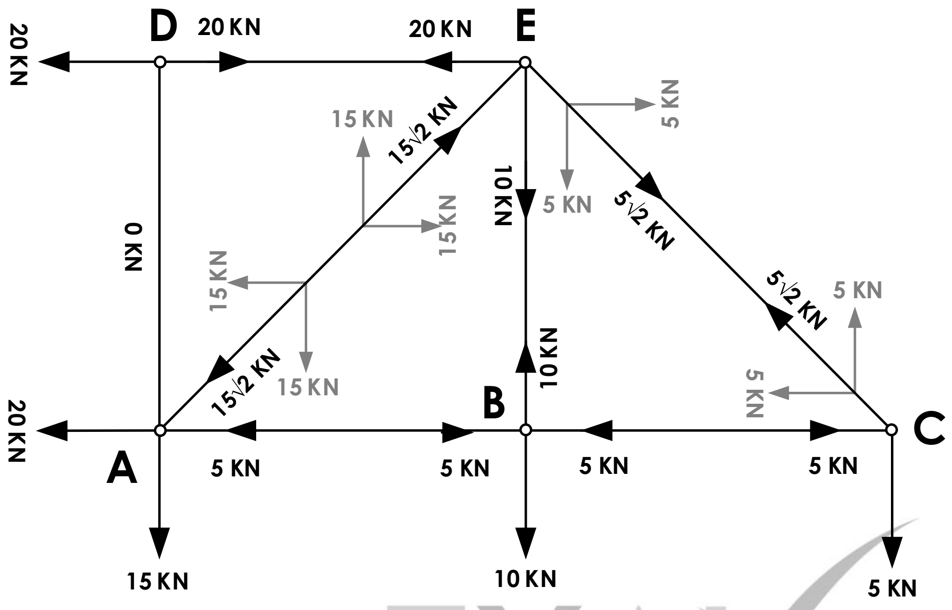
Environmental Engineering

"Education is the most Powerful Weapon which you can use to change the world."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

Solution



METHOD OF SECTION

Procedure

1. Find the support reactions,
2. Cut a section in the given truss in such a way that the member in which force is to be determined will be cutted,
3. Consider equilibrium of either RHS or LHS of the section and find the force in concerned member.

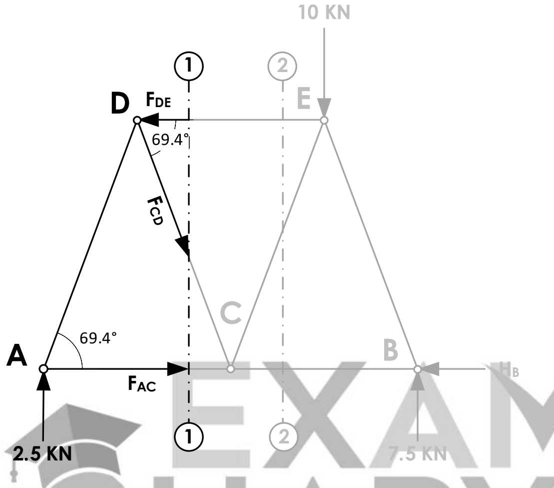
Note
- As far as possible don't cut more than three members since we have only 3 conditions or equilibrium conditions.

$$\sum V = 0$$

$$V_A + V_B - 10 = 0$$

$$V_A = 2.5 \text{ KN}$$

Considering equilibrium at section 1-1,

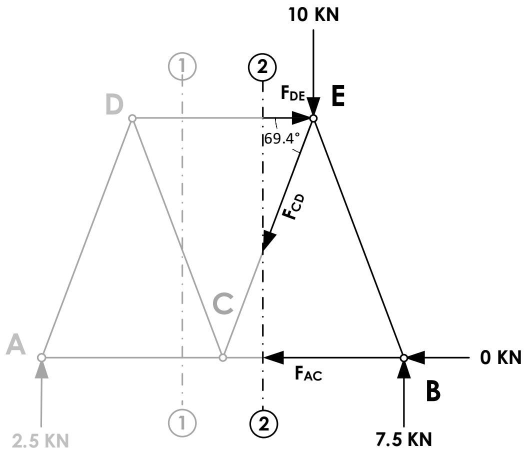


$$\sum V = 0$$

$$2.5 - F_{CD} \sin 69.44° = 0$$

$$F_{CD} = 2.67 \text{ KN (T)}$$

Now, considering equilibrium at section 2-2,



CHAPTER – 5

ENERGY THEOREMS

STRAIN ENERGY THEOREM

The energy stored in a body due to its straining (deformation) is known as strain energy of body. When an elastic member is deformed under the action of an external loading the member is said to have possessed or stored energy which is called the strain energy of the member or the resilience of the member. The strain energy stored by a member so deformed is equal to the amount of work done by the external forces to produce the deformation

RESILIENCE

The ability of material to store strain energy up to elastic limit.

PROOF RESILIENCE

It is the maximum strain energy that can be stored up to the elastic limit.

MODULUS OF PROOF RESILIENCE

The maximum strain energy that can be stored up to the elastic limit in unit volume of material.

TOUGHNESS

It is ability of material to stored strain energy up to fractured point.

MODULUS OF TOUGHNESS

It is the ability of material to stored strain energy in unit volume up to the fractured point.

External work done = Force × Displacement

$$= \frac{P\delta L}{2}$$

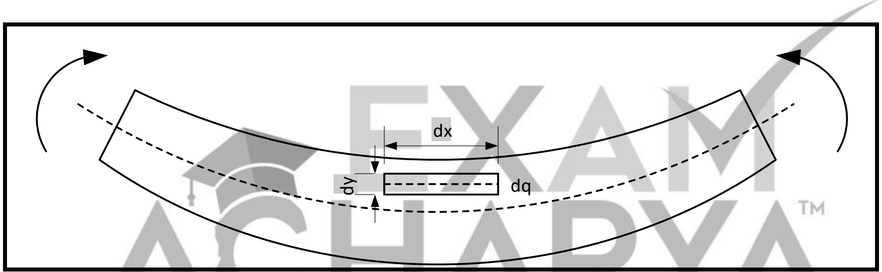
Internal work done, $U = \frac{1}{2}P\delta L$

$$= \frac{1}{2} \times P \times \frac{PL}{AE}$$

$$= \frac{1}{2}\frac{P^2LA}{A^2E}$$

$$= \frac{1}{2}\sigma^2\frac{V}{E}$$

$$\therefore \mathbf{U = \frac{1}{2} \times \sigma^2 \times \frac{Volume}{E}}$$



$$\sigma_x = \frac{M_x}{I_x} \times y_x$$

$$U_x = \frac{1}{2} \times \sigma_x^2 \times \frac{Volume}{E}$$

$$= \frac{1}{2E} \times \left(\frac{M_x}{I_x} \times y_x\right)^2 \times da \times dx$$

$$U = \int U_x$$

$$= \frac{1}{2E}\int \frac{M_X^2 dx}{I_X^2} \int y_x^2 da$$

$$= \frac{1}{2E}\frac{\int M_x^2 dx}{I^2} \times I \;\; [\because I = Ar^2]$$

$$= \frac{1}{2EI}\int M_X^2 dx$$

$$U = \frac{1}{2}P\Delta$$

Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
Total full length test	: 13
Mock test	: 16
Total test	: 80

As we know,

$$\frac{1}{2}P\Delta = \frac{1}{2EI}\int M_X^2 dx$$

$$\frac{1}{2}P\Delta = \frac{1}{2EI}\int_0^L (-Px)^2 dx$$

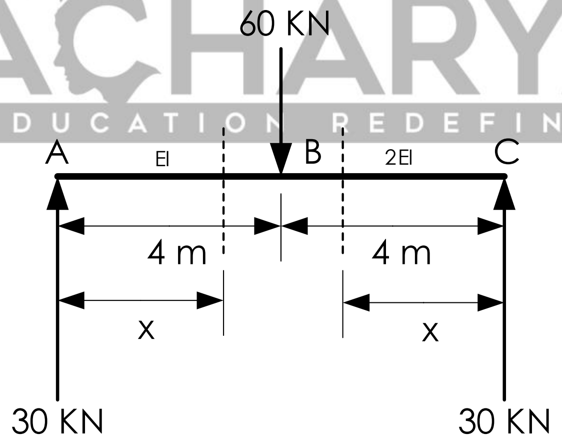
$$\Delta = \frac{P}{3EI}[x^3]_0^L$$

$$\Delta \frac{PL^3}{3EI}$$

Answer:

The deflection at free end is $\frac{PL^3}{3EI}$

Q2. Determine deflection under 60 KN load in beam shown in fig.



Solution

Member	Origin	Mx	Limit
AB	A	30x	0 – 4
BC	C	30x	0 – 4

As we know,

$$\frac{1}{2}P\Delta = \frac{1}{2EI}\int M_X^2 dx$$

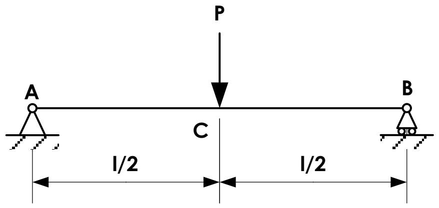
$$\frac{1}{2}\times 60\times\Delta = \frac{1}{2EI}\int_0^4 (30x)^2 dx + \frac{1}{2\times 2E\times I}\int_0^4 (30x)^2 dx$$

$$\Delta = \frac{27}{EI}$$

Answer:

The deflection at free end is $\frac{27}{EI}$

Q4. Determine the vertical displacement of point C from the fig. shown in fig below.



Solution

Member	Origin	Mx	Limit
AC	A	$\frac{Px}{2}$	$0 - \frac{l}{2}$
CB	C	$\frac{Px}{2}$	$0 - \frac{l}{2}$

As we know,

$$\frac{1}{2}P\Delta = 2 \times \left(\frac{1}{2EI}\int M_X^2 dx\right)$$

$$\frac{1}{2}P\Delta = 2 \times \left(\frac{1}{2EI}\int_0^{\frac{l}{2}} \left(\frac{Px}{2}\right)^2 dx\right)$$

$$\frac{1}{2}P\Delta = 2 \times \left(\frac{1}{2EI} \times \frac{P^2}{4} \times \left[\frac{x^3}{3}\right]_0^{\frac{l}{2}}\right)$$

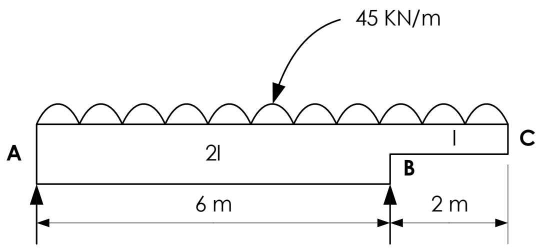
$$\Delta = \frac{Pl^3}{48EI}$$

Answer

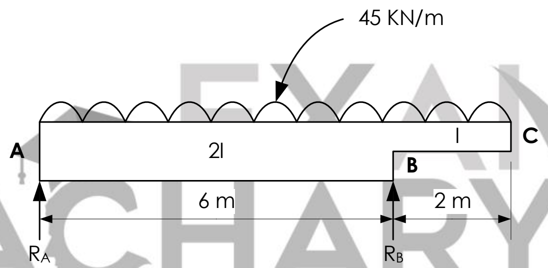
The vertical displacement at C is $\frac{Pl^3}{48EI}$

NUMERICAL

Q1. Find the deflection at the end of the overhanging beam.



Solution



Taking moment about A,

$$\sum M = 0$$

$$\left(45 \times 8 \times \frac{8}{2}\right) - (R_B \times 6) = 0$$

$$R_B = 240 \text{ KN}$$

Taking all the vertical forces,

$$\sum V = 0$$

$$R_A + R_B - (45 \times 8) = 0$$

$$R_A = 120 \text{ KN}$$

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EXAM ACHARYA
EDUCATION REDEFINED

Fluid Mechanics and Hydraulic Machines

"Success Consists of going from Failure without Loss of Enthusiasm."

Winston Churchill

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

$$= \int_0^6 \frac{\left(-40x^2 + \frac{15}{2}x^3\right)}{2EI} dx + \int_0^2 \frac{\left(\frac{45x^3}{2}\right)}{EI} dx$$

$$= \frac{20}{EI}\left[\frac{-x^3}{3}\right]_0^6 + \frac{15}{4EI}\left[\frac{x^4}{4}\right]_0^6 + \frac{45}{2EI}\left[\frac{x^4}{4}\right]_0^2$$

$$= \frac{-225 + 90}{EI}$$

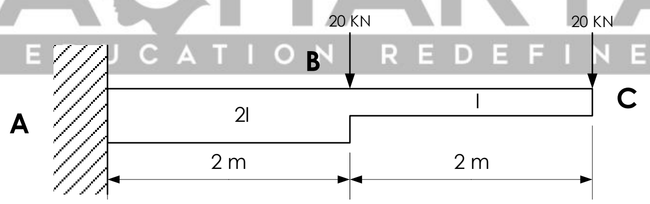
$$= -\frac{135}{EI}$$

$$= \frac{135}{EI} \text{ (upward)}$$

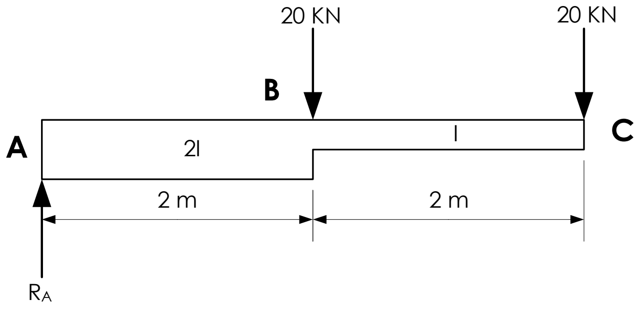
Answer

The vertical displacement at C is $\frac{135}{EI}$ upward.

Q2. Determine the vertical deflection of a cantilever beam shown in the fig.



Solution

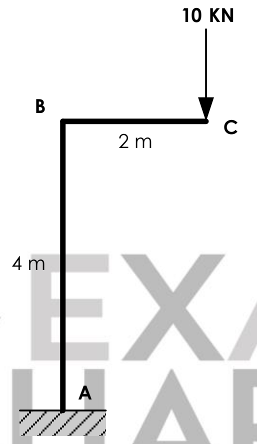


CASTIGLIANO'S THEOREM

$$\delta = \int M_x \frac{\frac{dM_x}{dW}}{EI}\, dx$$

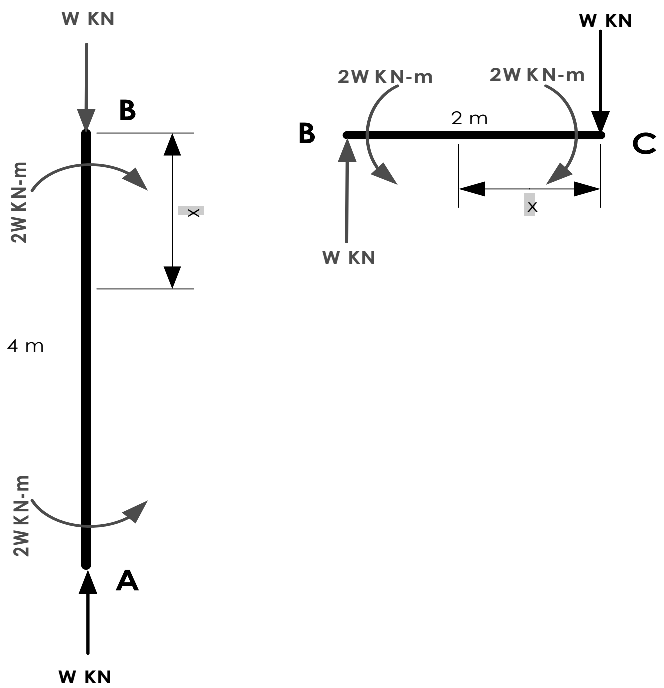
NUMERICAL

Find the vertical displacement of the joint C in the vertical frame.

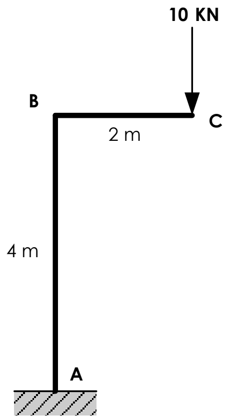


Solution

Free body diagram,

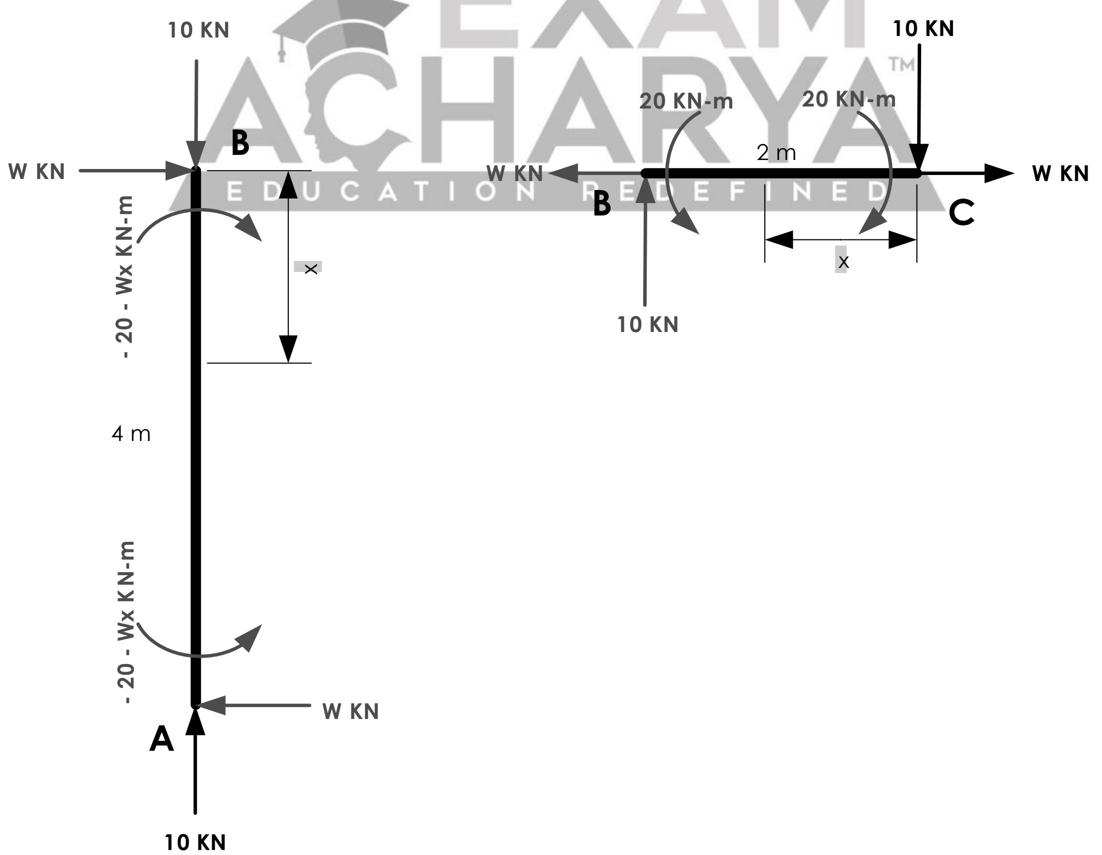


Q2. Find the horizontal displacement of the joint C in the vertical frame.



Solution

Free body diagram,



Here,

W = 0 KN

CASTIGLIANO'S 2ND THEOREM

The first partial derivative of strain energy with respect to applied load gives the displacement in the direction of applied load.

$$\boldsymbol{\delta = \frac{\partial U}{\partial W}}$$

ANALYSIS OF TRUSS BY CASTIGLIANO'S THEOREM

As we know,

$$\delta = \frac{\partial U}{\partial W}$$

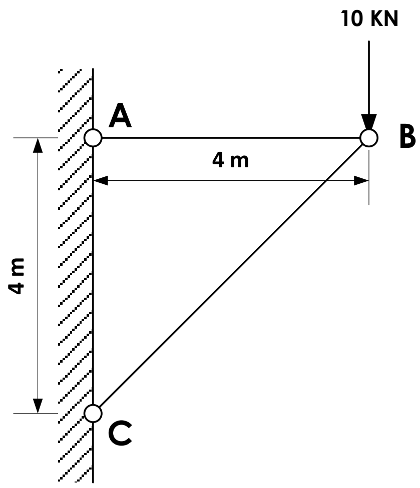
$$= \sum \frac{P \frac{\partial P}{\partial W} L}{AE}$$

$$= \frac{\sum PKL}{AE} \left[\text{where, } K = \frac{\partial P}{\partial W}\right]$$

$$\boldsymbol{\delta = \frac{\sum PKL}{AE}}$$

NUMERICAL

Q1. For the truss shown in fig. estimate the vertical deflection of joint B given that, cross sectional area of each member is 1550 m^2, Elastic constant (E) = $\mathbf{2 \times 10^5}$ N-mm^{-2}



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7622050066

Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
Total full length test	: 13
Mock test	: 16
Total test	: 80

$$\therefore \delta = \sum \frac{\text{PKl}}{\text{AE}}$$

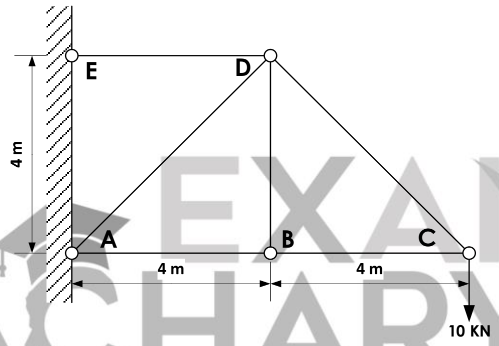
$$= 0.129 + 0.365 \text{ mm}$$

$$= 0.494 \text{ mm}$$

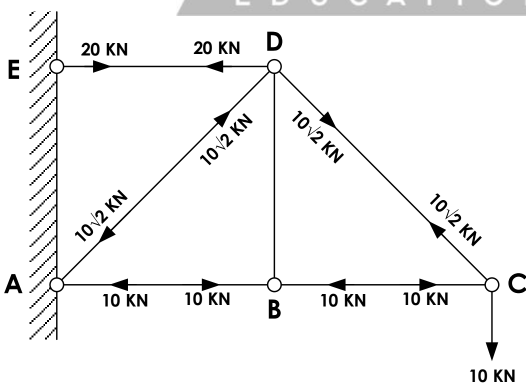
Answer

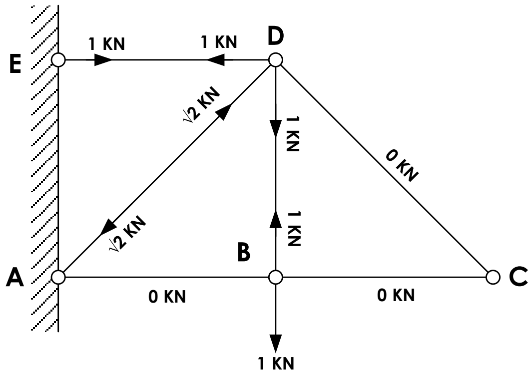
The vertical deflection at point B is 0.494 mm.

Q2. Find the vertical deflection of point B.



Solution



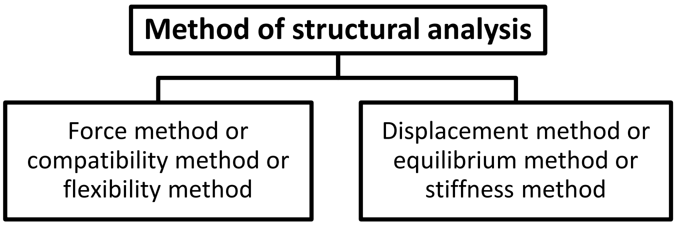


Member	P values	K values	l m	$\Delta = \frac{\text{PKl}}{\text{AE}}$
AB	-10	0	4	0
CD	$10\sqrt{2}$	0	$4\sqrt{2}$	0
BC	-10	0	4	0
BD	0	1	4	0
AD	$-10\sqrt{2}$	$-\sqrt{2}$	$4\sqrt{2}$	$80\sqrt{2}/\text{AE}$
ED	20	1	4	$80/\text{AE}$

CHAPTER – 6

METHOD OF STRUCTURAL ANALYSIS

METHOD OF STRUCTURAL ANALYSIS



COMPARISON BETWEEN FORCE AND DISPLACEMENT METHOD

Force method	Displacement method
In this method forces (loads and moments) are considered as unknown.	In this method displacements (deflection and rotation) are considered as unknown.
No. of compatibility equations required remains equal to no. of unknown forces.	No. of joint equilibrium conditions required remains equal to the no. of unknown displacements.
It is suitable when D_k is greater than Ds.	It is suitable when Ds is greater than D_k.
E.g.: Castigliano's theorem Unit load method Flexibility matrix method Mohr-coulomb's equation Three moment theorem etc.	E.g.: Moment distribution method Slope deflection method Stiffness matrix method Theorem of minimum potential energy etc.

MOMENT DISTRIBUTION METHOD

The moment distribution method is a structural analysis method for statically indeterminate beams and frames developed by Hardy Cross.

We know that,

$$EI\frac{d^2y}{dx^2} = Rx - M$$

Integrate both side with respect to x,

$$EI\frac{dy}{dx} = \frac{Rx^2}{2} - Mx + c_1 \text{ ....(i)}$$

$$\text{At } x = l, \frac{dy}{dx} = 0$$

$$\therefore 0 = \frac{Rl^2}{2} - Ml + c_1$$

$$c_1 = Ml - \frac{Rl^2}{2}$$

$$\therefore EI\frac{dy}{dx} = \frac{Rx^2}{2} - Mx + Ml - \frac{Rl^2}{2}$$

Again, integrate both side with respect to x,

$$EIy = \frac{Rx^3}{6} - \frac{Mx^2}{2} + Mlx - \frac{Rl^2x}{2} + c_2 \text{ ....(ii)}$$

$$\text{At } x = 0, y = 0$$

$$\therefore c_2 = 0$$

$$\therefore EIy = \frac{Rx^3}{6} - \frac{Mx^2}{2} + Mlx - \frac{Rl^2x}{2}$$

$$\text{at } x = l, y = 0$$

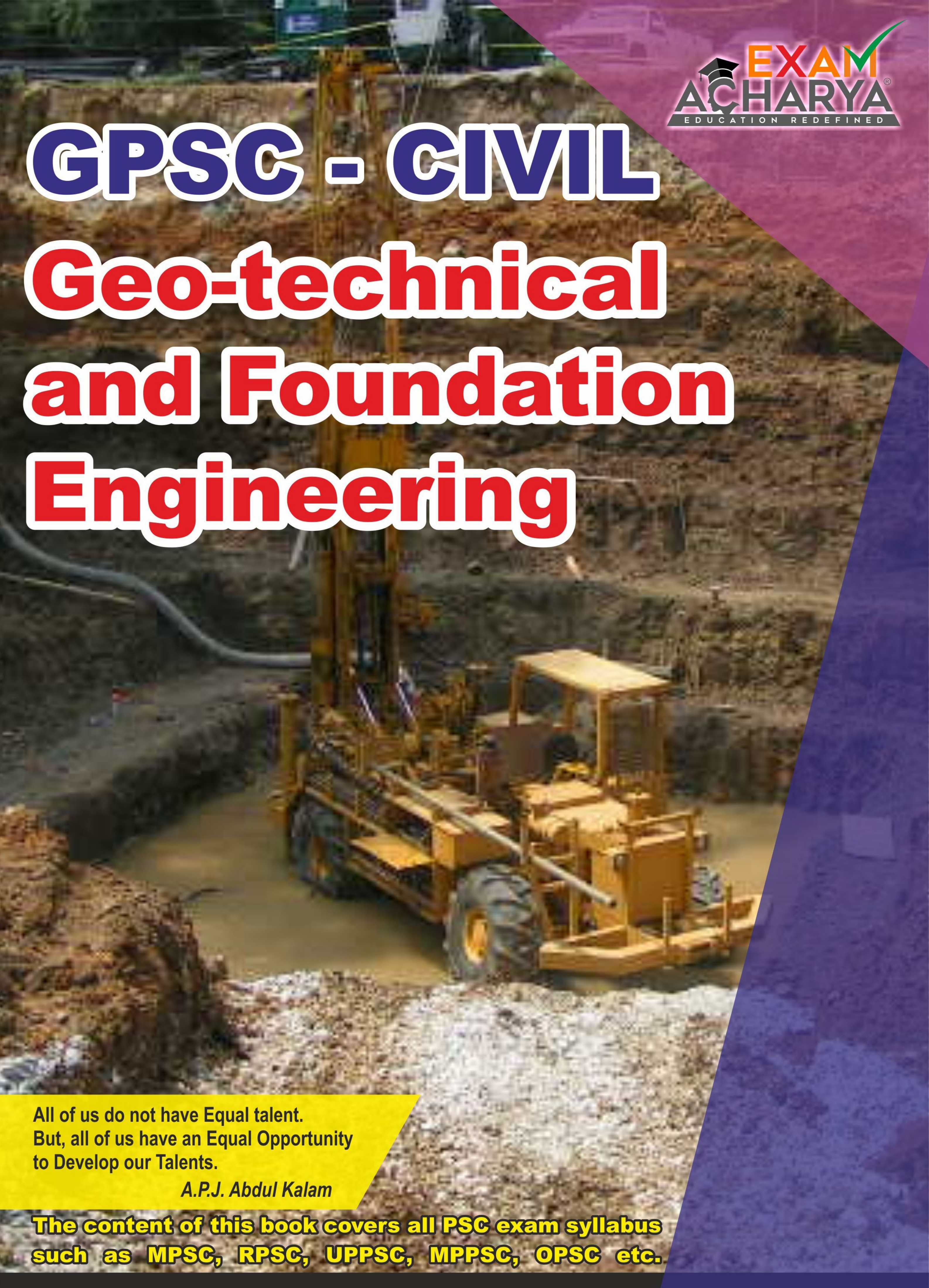
$$\therefore 0 = \frac{Rl^3}{6} - \frac{Ml^2}{2} + Ml^2 - \frac{Rl^3}{2}$$

$$\frac{1}{3}Rl^3 = \frac{1}{2}Ml^2$$

$$R = \frac{3M}{2l}$$

$$\because EI\frac{dy}{dx} = \frac{Rx^2}{2} - Mx + Ml - \frac{Rl^2}{2}$$

$$EI\frac{dy}{dx} = \frac{3Mx^2}{4l} - Mx + Ml - \frac{3Ml}{4}$$



EXAM ACHARYA
EDUCATION REDEFINED

GPSC - CIVIL

Geo-technical and Foundation Engineering

All of us do not have Equal talent.
But, all of us have an Equal Opportunity
to Develop our Talents.

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

$$\therefore 0 = \frac{Rl^3}{6} - \frac{Ml^2}{2} + c_1 l$$

$$c_1 = \frac{Ml}{2} - \frac{Rl^2}{6}$$

$$\therefore EI\frac{dy}{dx} = \frac{Rx^2}{2} - Mx + \frac{Ml}{2} - \frac{Rl^2}{6}$$

$$\therefore EIy = \frac{Rx^3}{6} - \frac{Mx^2}{2} + \frac{Mlx}{2} - \frac{Rl^2x}{6}$$

$$R = \frac{M}{l}$$

$$\text{at } x = 0, \frac{dy}{dx} = \theta_B$$

$$\therefore EI\theta_B = \frac{Ml}{2} - \frac{Rl^2}{6}$$

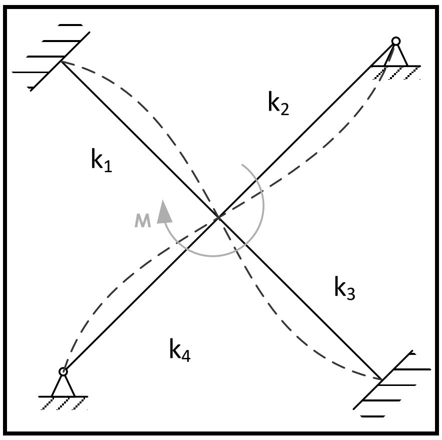
$$EI\theta_B = \frac{Ml}{2} - \frac{Ml}{6} = \frac{Ml}{3}$$

$$\frac{M}{\theta_B} = \frac{3EI}{l} = K$$

$$\mathbf{K = \frac{3EI}{l}}$$

DISTRIBUTION FACTOR

It is define as the ratio in which the applied moment on a joint is being shared by all the members connecting on that joint.

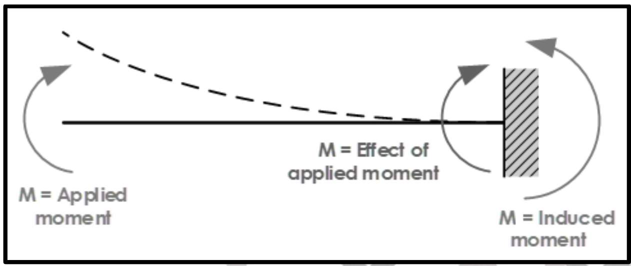


$$COF = \frac{\text{induced moment}}{\text{applied moment}}$$

$$= \frac{\frac{M}{2}}{M}$$

$$= \frac{1}{2}$$

Case – 2

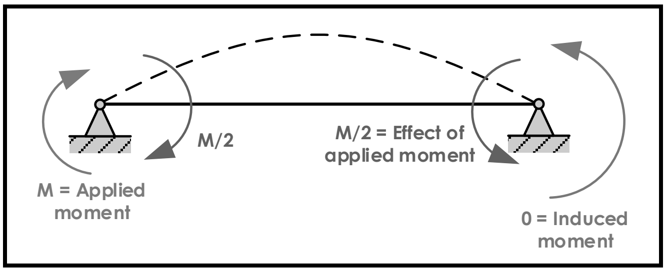


$$COF = \frac{\text{induced moment}}{\text{applied moment}}$$

$$= \frac{-M}{M}$$

$$= -1$$

Case – 3



$$COF = \frac{\text{induced moment}}{\text{applied moment}}$$

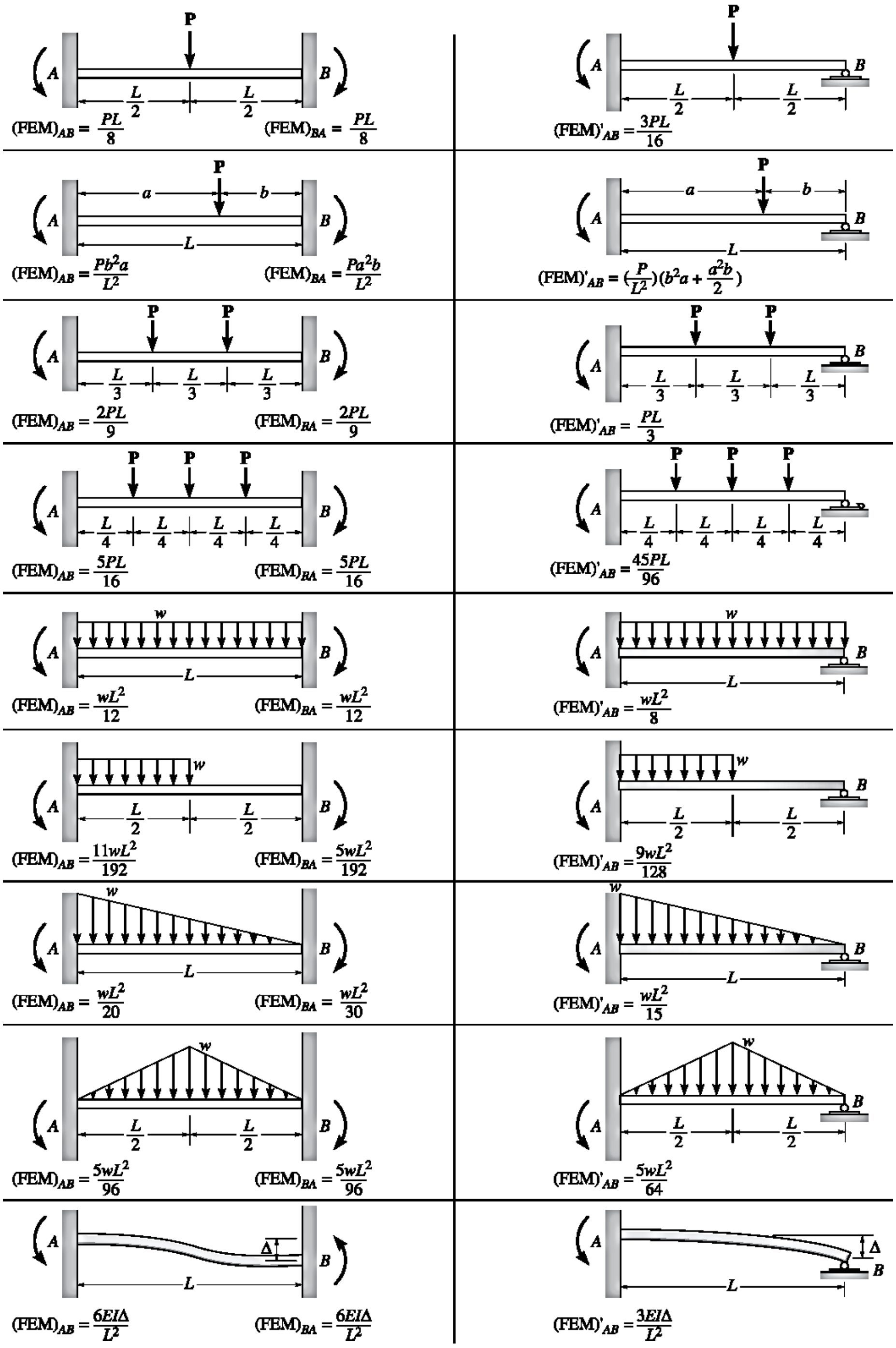
$$= \frac{0}{M}$$

$$= 0$$

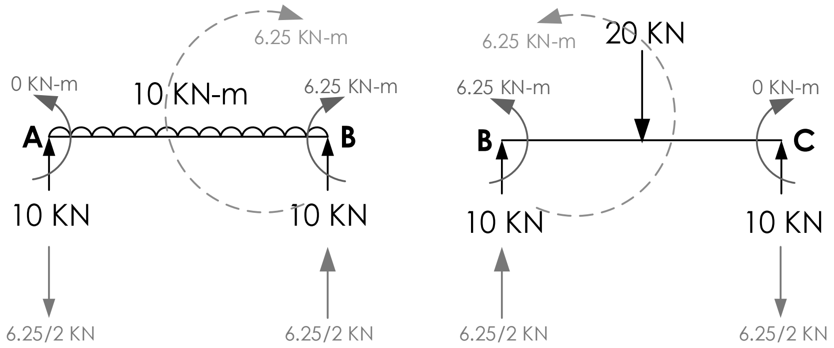
$$M^F l = \frac{1}{2} \times l \times \frac{Pl}{4}$$

$$M^F = \frac{Pl}{8}$$

FIXED END MOMENT FOR DIFFERENT LOADING CONDITIONS



Determination of support reactions,

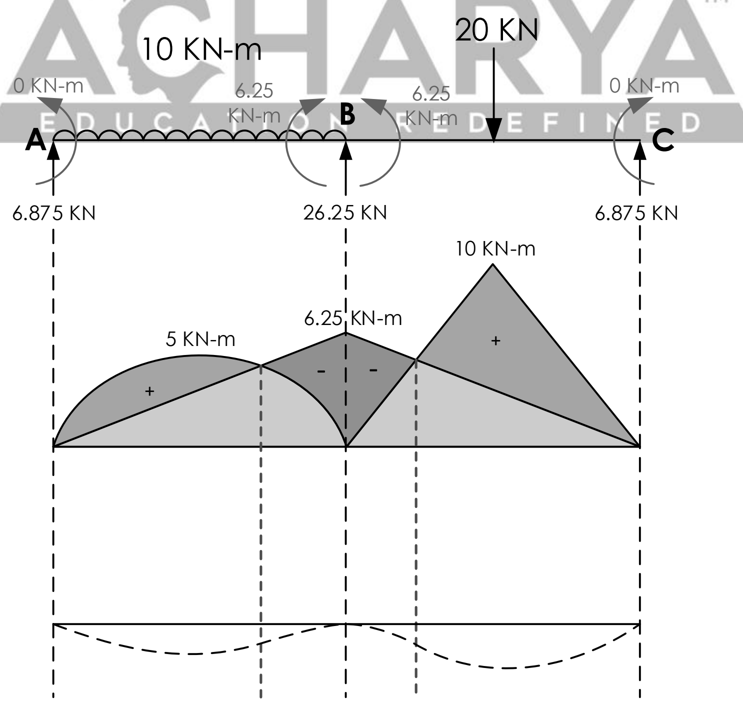


$$R_A = 10 - \frac{6.25}{2}\ KN = 6.875\ KN\ (\uparrow)$$

$$R_B = \left(10 + \frac{6.25}{2}\right) + \left(10 + \frac{6.25}{2}\right) KN = 26.25\ KN\ (\uparrow)$$

$$R_C = 10 - \frac{6.625}{2}\ KN = 6.875\ KN\ (\uparrow)$$

BMD and deflected shape,



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Test Series Available..

Total weekly test	***: 35***
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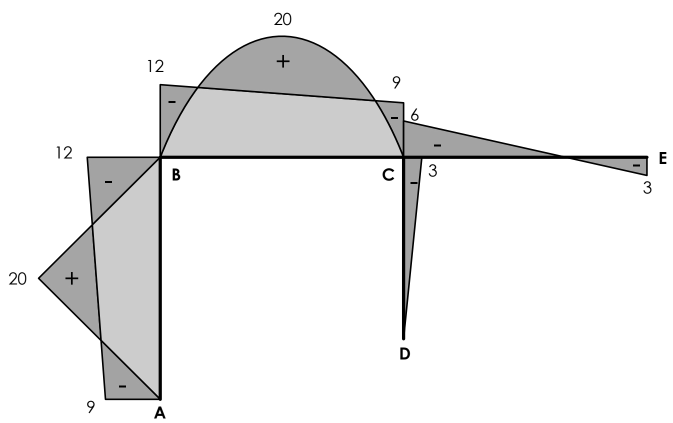
Mock test	***: 16***
Total test	***: 80***

Moment distribution,

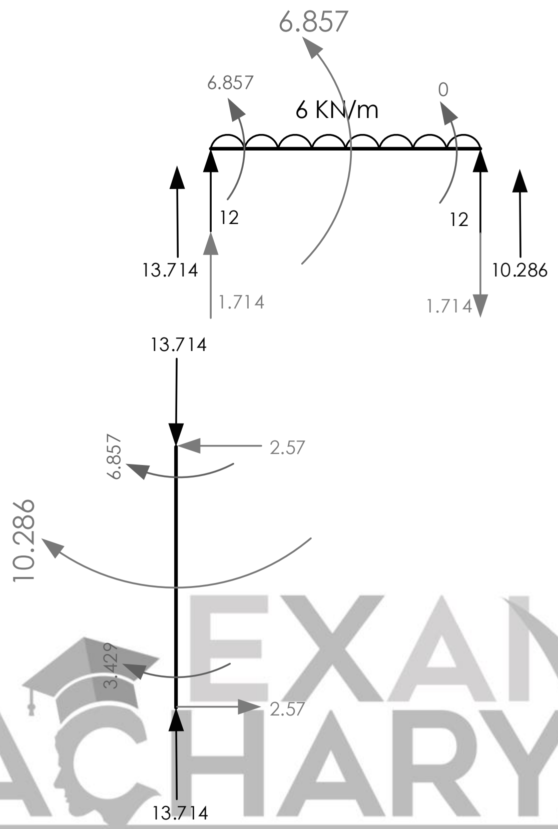
Joint	A	B		C			D
Member	AB	BA	BC	CB	CE	CD	DC
Distribution factor		0.33	0.67	0.40	0.40	0.20	
Fixed end moment	-10.00	10.00	-13.33	13.33	0.00	0.00	0.00
Balance		1.111	2.222	-5.333	-5.333	-2.667	
Carry over	0.556		-2.667	1.111			-1.333
Balance		0.889	1.778	-0.444	-0.444	-0.222	
Carry over	0.444		-0.222	0.889			-0.111
Balance		0.074	0.148	-0.356	-0.356	-0.178	
Carry over	0.037		-0.178	0.074			-0.089
Balance		0.059	0.119	-0.030	-0.030	-0.015	
Carry over	0.030		-0.015	0.059			-0.007
Balance and CO	0.002	0.005	0.010	-0.024	-0.024	-0.012	-0.006
Final moments	-9	12	-12	9	-6	-3	-2

$$M_{EC} = \frac{1}{2} M_{CE} = \frac{1}{2} \times (-6) \text{ KN-m} = -3 \text{ KN-m}$$

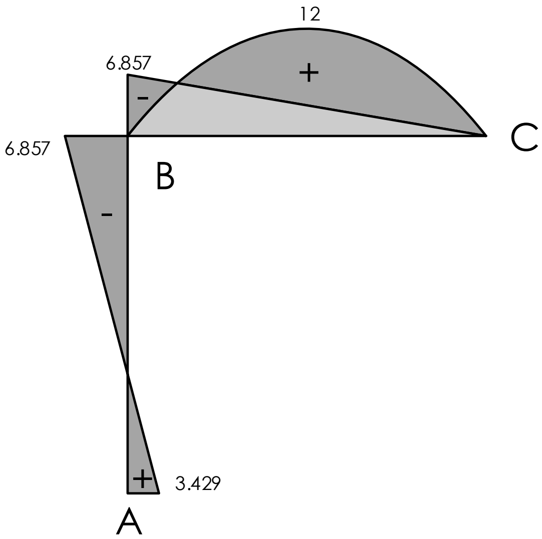
Bending moment diagram,



Determination of support reaction,



Bending moment diagram,

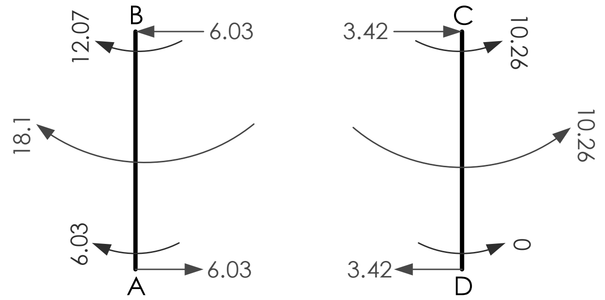


Joint	Member	K	$\sum k$	Distribution factor
B	BA	$\frac{4EI}{3}$	$\frac{7EI}{3}$	$\frac{4}{7}$
	BC	EI		$\frac{3}{7}$
C	CB	EI	2EI	$\frac{1}{2}$
	CD	EI		$\frac{1}{2}$

Moment distribution,

Joints	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
distribution factor		0.57	0.43	0.50	0.50	
Fixed end moment	0.00	0.00	-16.00	16.00	0.00	0.00
Balance		9.14	6.86	-8.00	-8.00	
Carry over	4.57		-4.00	3.43		
Balance		2.29	1.71	-1.71	-1.71	
Carry over	1.14		-0.86	0.86		
Balance		0.49	0.37	-0.43	-0.43	
Carry over	0.24		-0.21	0.18		
Balance		0.12	0.09	-0.09	-0.09	
Carry over	0.06		-0.05	0.05		
Balance and carry over	0.01	0.03	0.02	-0.02	-0.02	
Final moment	6.03	12.07	-12.07	10.26	-10.26	0.00

Determination of sway force,



GPSC - CIVIL

Reinforced Cement Concrete

Education's purpose is to
replace an empty mind with an open one.

Malcolm Forbes

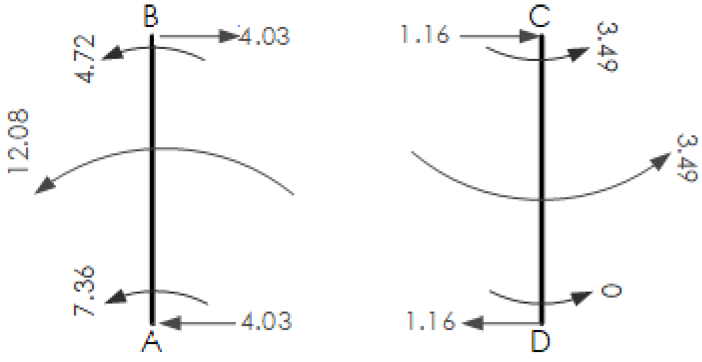
The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

EXAM ACHARYA
EDUCATION REDEFINED

Moment distribution,

Joints	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
distribution factor		0.57	0.43	0.50	0.50	
Sway moment	-10	-10	0	0	-5	0
Balance		5.71	4.29	2.50	2.50	
Carry over	2.86		1.25	2.14		
Balance		-0.71	-0.54	-1.07	-1.07	
Carry over	-0.36		-0.54	-0.27		
Balance		0.31	0.23	0.13	0.13	
Carry over	0.15		0.07	0.11		
Balance		-0.04	-0.03	-0.06	-0.06	
Carry over	-0.02		-0.03	-0.01		
Balance and carry over	0.01	0.02	0.01	0.01	0.01	
Final moment	-7.36	-4.72	4.72	3.49	-3.49	0.00

Sway force on beam level due to assumed moments,



Sway force at beam level due to assumed sway moment, S_{cal} = 4.03 + 1.16 KN

= 5.19 KN (→)

Correction factor = $\frac{2.61}{5.19}$

= 0.503

$$V_D = 24.58 \text{ KN},$$

$$H_D = 4 \text{ KN}.$$

Slope Deflection Method

It was given by George A. Maney in 1914. He also gave the concept of carry over moment. It is a type of displacement method. Equilibrium of joints is considered to solve the kinematic indeterminacy of structure.

Assumptions

1. Axially forces and axial deformation are neglected,
2. Clockwise end moment is consider as positive,
3. Anticlockwise end moment is consider as negative,
4. Sagging bending moment is consider as positive,
5. Hogging bending moment is consider as negative,
6. Member can't fail by buckling, and it can only fail in bending.

GENERATION OF SLOPE DEFLECTION EQUATION

Slope deflection equation is generated by superimposition of all the effects (of moments) at the concurred end.

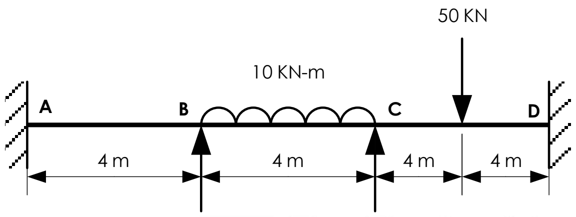
Let us consider that there is a span AB which is continue over both the ends and is subjected to a load system as shown in fig.

The above equations are derived for a span which is supported on both ends. Hence these equations can't be applied for cantilever.

If left support goes upward then δ is taken to be positive (+ve) or if right support goes down, then also δ is taken to be positive (+ve).

NUMERICAL

Q1. Analyze the beam shown in figure below,



Solution

Fixed end moments,

$$M_{AB}^{F} = M_{BA}^{F} = 0 \text{ [as there is no load in AB span]}$$

$$M_{BC}^{F} = -\frac{wl^2}{12} = -\frac{10 \times 4^2}{12} = -13.3 \text{ KNm}$$

$$M_{CB}^{F} = \frac{wl^2}{12} = \frac{10 \times 4^2}{12} = 13.3 \text{ KNm}$$

$$M_{CD}^{F} = -\frac{Wb^2a}{l^2} = -\frac{50 \times 2^2 \times 2}{4^2} = -25 \text{ kNm}$$

$$M_{DC}^{F} = \frac{Wa^2b}{l^2} = \frac{50 \times 2^2 \times 2}{4^2} = 25 \text{ kNm}$$

Slope deflection equation,

$$M_{AB} = -M_{AB}^{F} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}[0 + \theta_B - 0]$$

$$= \frac{EI\theta_B}{2}$$

$$EI\theta_B + EI\theta_B + \frac{EI\theta_C}{2} - 13.33 = 0$$

$$2EI\theta_B + \frac{EI\theta_C}{2} = 13.33 \text{ ....(i)}$$

again,

$$\sum M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$13.33 + EI\theta_C + \frac{EI\theta_B}{2} - 25 + EI\theta_C = 0$$

$$2EI\theta_C + \frac{EI\theta_B}{2} = 11.67 \text{ ....(ii)}$$

From equation no. i and ii we get,

$$\theta_B = \frac{5.55}{EI}$$

$$\theta_C = \frac{4.44}{EI}$$

Now, putting the value of θ_B and θ_C we get,

$$M_{AB} = 2.78 \text{ KNm},$$

$$M_{BA} = 5.56 \text{ KNm},$$

$$M_{BC} = -5.56 \text{ KNm},$$

$$M_{CB} = 20.56 \text{ KNm},$$

$$M_{CD} = -20.56 \text{ KNm},$$

$$M_{DC} = 27.22 \text{ KNm}.$$

Bending moment diagram,

Solution

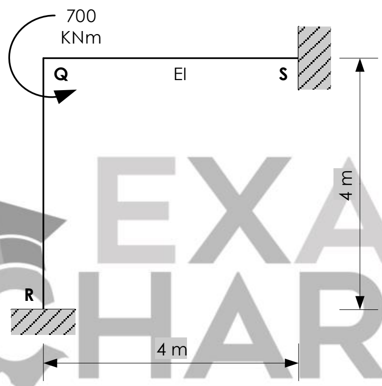
Replace all the loadings by applying a moment at Q,

$$\sum M = 0$$

$$\left(1650 \times 2 \times \frac{2}{2}\right) - (2000 \times 2) - M_Q = 0$$

$$M_Q = -700 \text{ KNm}$$

New diagram,



As we know,

$$\sum M_Q = 0$$

$$M_{QR} + M_{QS} = -700$$

$$EI\theta_Q + EI\theta_Q = -700$$

$$2EI\theta_Q = -700$$

$$\theta_Q = -\frac{350}{EI}$$

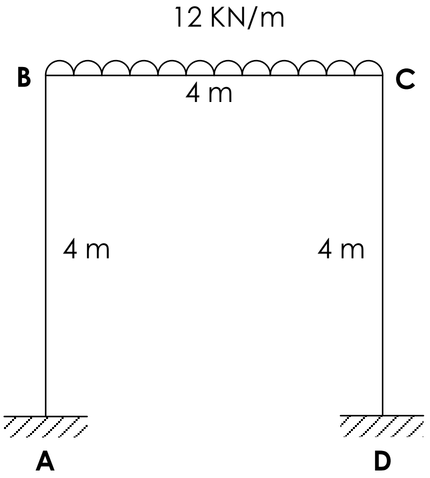
$$= -\frac{350}{2.5 \times 10^4 \times 8 \times 10^8 \times 10^{-9}}$$

$$= -0.0175 \text{ rad}$$

$$= -1.0027°$$

$$= 1.0027° \text{ (anticlockwise)}$$

Q4. Analyze the frame shown in fig. by slope deflection method,



Solution

Fixed end moments,

$$M_{AB}^F = M_{BA}^F = M_{CD}^F = M_{DC}^F = 0 \text{ KN-m}$$

$$M_{BC}^F = -\frac{12 \times 4^2}{12} = -16 \text{ KN-m}$$

$$M_{CB}^F = \frac{12 \times 4^2}{12} = 16 \text{ KN-m}$$

Slope deflection equation,

$$M_{AB} = \frac{EI\theta_B}{2}$$

$$M_{BA} = EI\theta_B$$

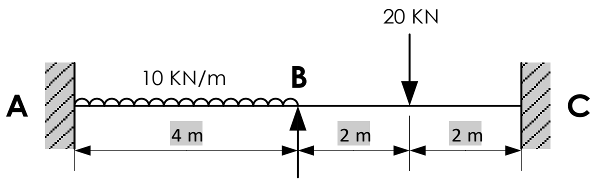
$$M_{BC} = -16 + EI\theta_B + \frac{EI\theta_C}{2}$$

$$M_{CB} = 16 + \frac{EI\theta_B}{2} + EI\theta_C$$

$$M_{CD} = EI\theta_C$$

$$M_{DC} = \frac{EI\theta_C}{2}$$

Q5. Analyze the frame shown in fig. by Maney's method, where, $E = 2 \times 10^4$ N/mm^2, $I = 3.6 \times 10^7$ mm^4. B sinks by 15 mm.



Solution

Fixed end moment,

$$M_{AB}^{F} = -\frac{10\times4^2}{12} = -13.33 \text{ KN-m}$$

$$M_{BA}^{F} = \frac{10\times4^2}{12} = 13.33 \text{ KN-m}$$

$$M_{BC}^{F} = -\frac{20\times4}{8} = -10 \text{ KN-m}$$

$$M_{CB}^{F} = \frac{20\times4}{8} = 10 \text{ KN-m}$$

Slope deflection equation,

$$M_{AB} = -13.333 + \frac{EI\theta_B}{2} - \left(\frac{3\times0.015}{4}\times\frac{7200}{2}\right)$$

$$= -53.83 + \frac{EI\theta_B}{2}$$

$$M_{BA} = 13.333 + EI\theta_B - \left(\frac{3\times0.015}{4}\times\frac{7200}{2}\right)$$

$$= -27.17 + EI\theta_B$$

$$M_{BC} = -10 + EI\theta_B - \left[\frac{3\times(-0.015)}{4}\times\frac{7200}{2}\right]$$

$$= 30.5 + EI\theta_B$$

$$M_{CB} = 10 + \frac{EI\theta_B}{2} - \left[\frac{3\times(-0.015)}{4}\times\frac{7200}{2}\right]$$

$$= 50.5 + \frac{EI\theta_B}{2}$$

EXAM ACHARYA

GPSC - CIVIL

Solid Mechanics

"Education is the most Powerful Weapon which you can use to change the world."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

GPSC - CIVIL
Solid Mechanics

Slope deflection equation,

$$M_{AB} = M_{AB}^{F} + \frac{2EI}{l}[2\theta_A + \theta_B]$$

$$= 0 + \frac{2EI}{4}[\theta_B]$$

$$= \frac{EI\theta_B}{2}$$

$$M_{BA} = M_{BA}^{F} + \frac{2EI}{l}[2\theta_B + \theta_A]$$

$$= 0 + \frac{2EI}{4}[2\theta_B]$$

$$= EI\theta_B$$

$$M_{BC} = M_{BC}^{F} + \frac{2EI}{l}[2\theta_B + \theta_C]$$

$$= -8 + \frac{2EI}{4}[2\theta_B + \theta_C]$$

$$= EI\theta_B + \frac{EI\theta_C}{2} - 8$$

$$M_{CB} = M_{CB}^{F} + \frac{2EI}{l}[2\theta_C + \theta_B]$$

$$= 8 + \frac{2EI}{4}[2\theta_C + \theta_B]$$

$$= EI\theta_C + \frac{EI\theta_B}{2} + 8$$

Joint equilibrium equation,

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B + EI\theta_B + \frac{EI\theta_C}{2} - 8 = 0$$

$$2EI\theta_B + \frac{EI\theta_C}{2} = 8 \text{ ....(i)}$$

CHAPTER – 7

ARCHES

Arches are structures composed of curvilinear members resting on supports. They are used for large-span structures, such as airplane hangars and long-span bridges. One of the main distinguishing features of an arch is the development of horizontal thrusts at the supports as well as the vertical reactions, even in the absence of a horizontal load. The internal forces at any section of an arch include axial compression, shearing force, and bending moment. The bending moment and shearing force at such section of an arch are comparatively smaller than those of a beam of the same span due to the presence of the horizontal thrusts. The horizontal thrusts significantly reduce the moments and shear forces at any section of the arch, which results in reduced member size and a more economical design compared to other structures. Additionally, arches are also aesthetically more pleasant than most structures.

Arches can be classified into following types,

a) Two-hinged arches,
b) Three-hinged arches,
c) Fixed arches.

The arches are funicular shape. Funicular shape is defined as the shape of thread which is hold on its end and is suspended due to its self-weight.

Any civil engineering structure which is in the funicular shape is known as the funicular structure. e.g. three hinge arches, two hinge arches, suspension cable, Intermediate spans of masonry bridge etc.

THREE-HINGED ARCHES

It is a type of inverted funicular structure which remains supported on both ends with hinged supports and also has a hinge on its crown. Both the supports can be or can't be on the same level.

Taking all the vertical forces,

$$\sum V = 0$$

$$V_A + V_B - W = 0$$

$$V_A = W - \frac{Wa}{L}$$

$$= \frac{W(L - a)}{L}$$

Taking moment about C,

$$\sum M = 0$$

$$(H_B \times h) - \left(V_B \times \frac{L}{2}\right) = 0$$

$$H_B = \frac{Wa}{2h}$$

Taking all the horizontal forces,

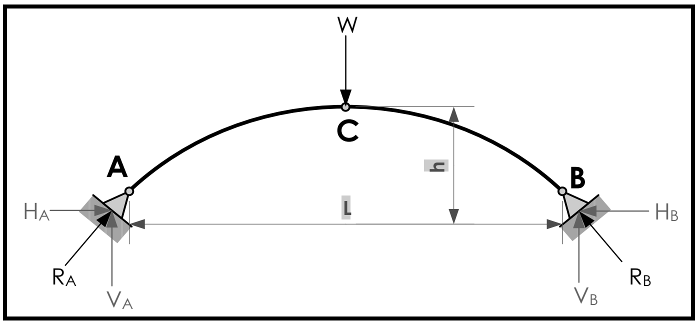
$$\sum H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = \frac{Wa}{2h}$$

All the four reaction can be easily found by four equation of equilibrium.

Example – 2



Here, $a = \frac{L}{2}$

$$(H_B \times h) - \left(\frac{wl}{2} \times \frac{l}{2}\right) + \left(w \times \frac{l}{2} \times \frac{l}{4}\right) = 0$$

$$H_B = \frac{wl^2}{8h}$$

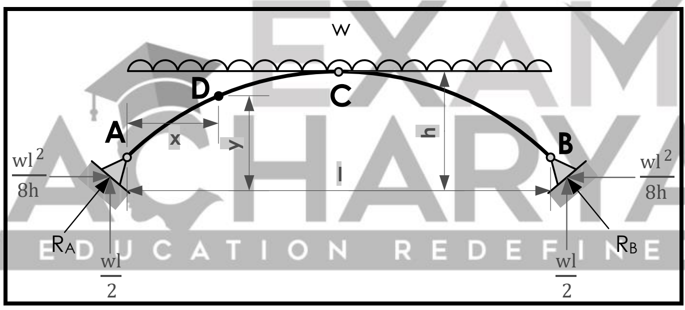
Taking all the horizontal forces,

$$\sum H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = \frac{wl^2}{8h}$$

Equation of Parabolic Arch



Taking moment at D,

$$M = \frac{wlx}{2} - \frac{wl^2y}{8h} - \frac{wx^2}{2}$$

$$M = \frac{wx}{2}(l - x) - \frac{wl^2y}{8h}$$

Just left to C, M = 0,

$$\therefore \frac{wx}{2}(l - x) - \frac{wl^2y}{8h} = 0$$

$$\frac{wx}{2}(l - x) = \frac{wl^2y}{8h}$$

$$y = \frac{4hx}{l^2}(l - x)$$

$$\mathbf{y = \frac{4hx}{l^2}(l - x)}$$

Taking all the vertical forces,

$$\sum V = 0$$

$$V_A + V_B - (50 \times 10) = 0$$

$$V_A = 500 - 125 \text{ KM} = 375 \text{ KN}$$

Taking moment about C,

$$\sum M = 0$$

$$4H_B - 10V_B = 0$$

$$H_B = \frac{10 \times 125}{4} \text{ KN} = 312.5 \text{ KN}$$

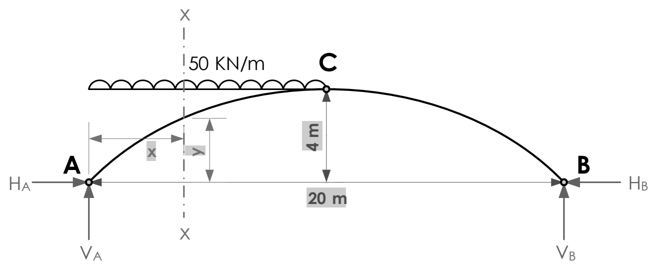
Taking all the horizontal forces,

$$\sum H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = 312.5 \text{ KN}$$

Calculation for maximum positive bending moment,



$$M_{x-x} = H_B y - V_B x$$

$$= 312.5(0.8x - 0.04x^2) - 125x$$

$$= 12.5x^2 - 125x$$

For maximum negative bending moment,

$$\frac{dM_{x-x}}{dx} = 0$$

$$25x - 125 = 0$$

$$x = 5\text{ m}$$

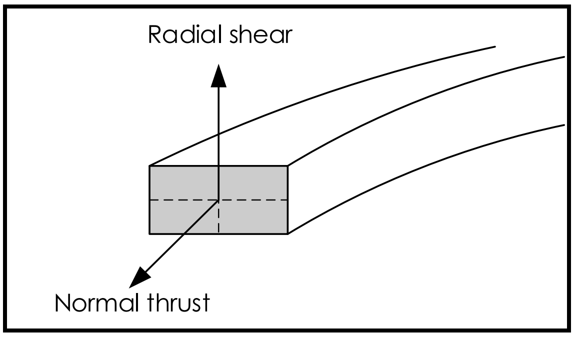
∴ Maximum negative bending moment,

$$M = (12.5 \times 5^2) - (125 \times 5)\text{ KN-m}$$

$$= -312.5\text{ KN-m}$$

NORMAL THRUST AND RADIAL SHEAR

Total force acting along the normal is called normal thrust and total force acting along the radial direction is called radial shear.



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7622050066

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Total weekly test : 35

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Total full length test : 13

Mock test : 16

Total test : 80

Taking moment about C,

$$\sum M_C = 0$$

$$(H_B \times 4) - \left(V_B \times \frac{20}{2}\right) = 0$$

$$H_B = 2\ KN$$

Taking all the horizontal forces,

$$\sum H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = 2\ KN$$

Calculation for reaction at supports,

$$R_A = \sqrt{V_A^2 + H_A^2}$$

$$= \sqrt{3.2^2 + 2^2}\ KN$$

$$= 3.77\ KN$$

Inclination with the horizontal,

$$\tan\theta = \frac{V_A}{H_A} = \frac{3.2}{2}$$

$$\theta_A = 57°59'$$

$$R_B = \sqrt{V_B^2 + H_B^2}$$

$$= \sqrt{0.8^2 + 2^2}\ KN$$

$$= 2.15\ KN$$

Inclination with the horizontal,

$$\tan\theta = \frac{V_A}{H_A} = \frac{0.8}{2}$$

$$\theta_A = 21°48'$$

GPSC - CIVIL

Surveying

The best Brains of the Nation may be found on the last Benches of the Classroom.

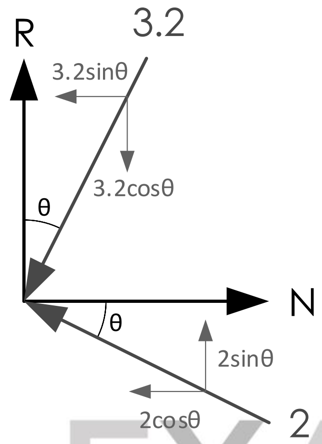
A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

At, x = 4

$$\tan\theta = \frac{dy}{dx} = \frac{20}{25} - \frac{20 \times 4}{25}$$

$$\theta = 25°38'$$



$$\sum V = 0$$

$$R - 3.2\cos\theta + 2\sin\theta = 0$$

$$R = 2.02 \text{ KN}$$

$$\sum H = 0$$

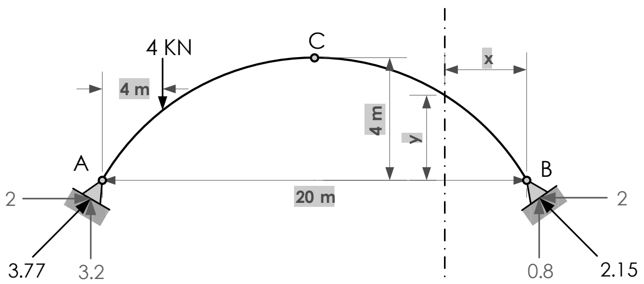
$$2\cos\theta + 3.2\sin\theta - N = 0$$

$$N = 3.19 \text{ KN}$$

Maximum positive bending moment is under load,

$$BM_{max}(+ve) = 7.68 \text{ KN-m}$$

Maximum negative bending moment lie somewhere in BC portion,



Vertical reactions can be determined through equilibrium conditions.

Taking moment about A,

$$\sum M = 0$$

$$Wa - V_B l = 0$$

$$V_B = \frac{Wa}{l}$$

Taking all the vertical forces,

$$\sum V = 0$$

$$V_A + V_B - W = 0$$

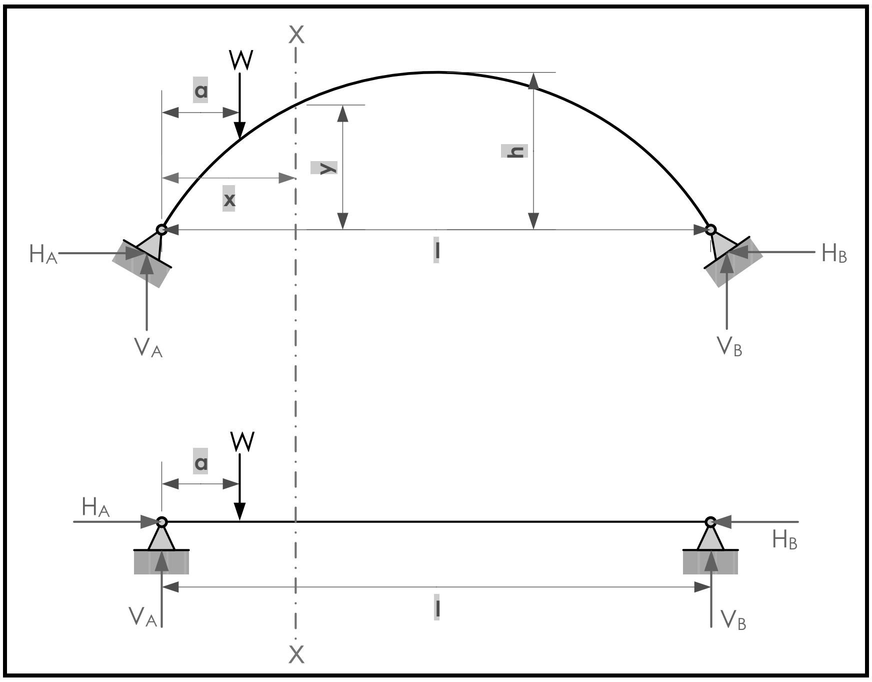
$$V_A = \frac{W(l - a)}{l}$$

Taking all the horizontal forces,

$$\sum H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = H \text{ (assume)}$$

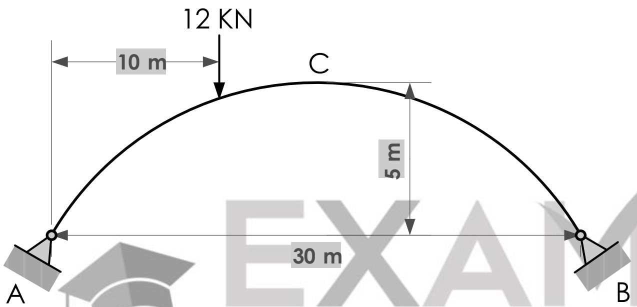


If EI is constant for whole the span,

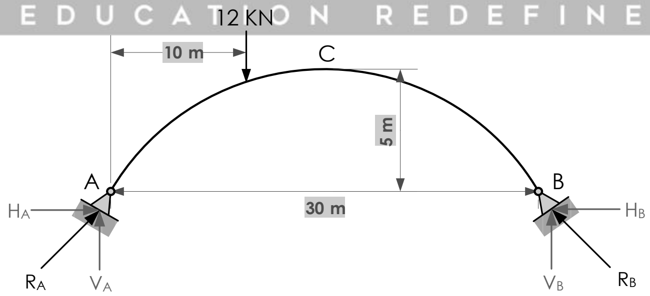
$$H = \frac{\int M' y \, dx}{\int y^2 \, dx}$$

NUMERICAL

Q1. A parabolic arch hinged at the ends has a span 30 m and rise 5 m. A concentrated load of 12 KN acts at 10 m from the left hinge. Calculate reaction at supports.



Solution



Taking moment about A,

$$\sum M_A = 0$$

$$(10 \times 12) - (V_B \times 30) = 0$$

$$V_B = 4 \text{ KN}$$

Taking all the vertical forces,

$$\sum V = 0$$

$$V_A + V_B - 12 = 0$$

$$= \frac{\int_0^{10}\left[8x\times\frac{x}{45}(30-x)\right]dx+\int_{10}^{30}\left[4(30-x)\times\frac{x}{45}(30-x)\right]dx}{\left\{\frac{x}{45}(30-x)\right\}^2}\ \text{KN}$$

$$= \frac{\frac{44000}{9}}{400}\ \text{KN}$$

$$= 12.22\ \text{KN}$$

Calculation for reaction,

$$R_A = \sqrt{V_A^2 + H_A^2}$$

$$= \sqrt{8^2 + 12.22^2}\ \text{KN}$$

$$= 14.61\ \text{KN}$$

$$\tan\theta = \frac{V_A}{H_A} = \frac{8}{12.22}$$

$$\theta = 33°14'$$

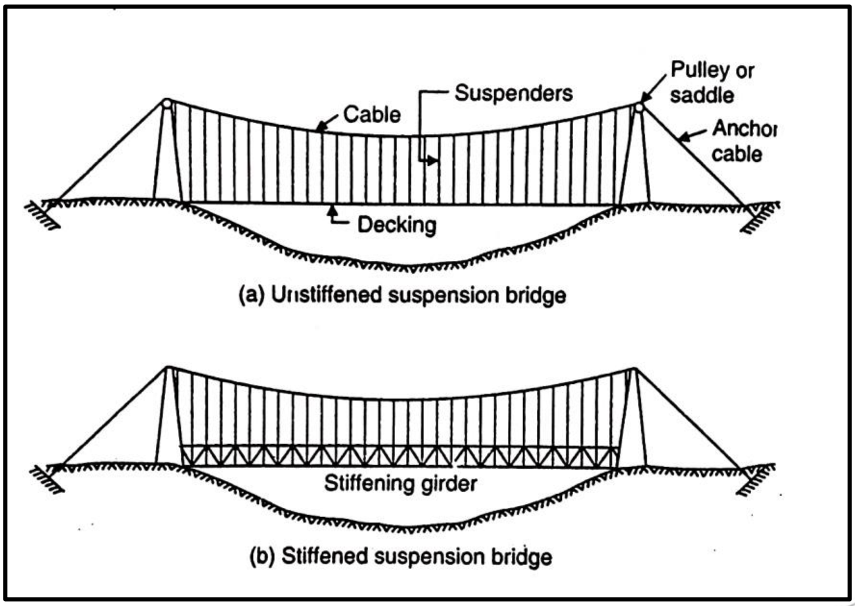
$$R_B = \sqrt{V_B^2 + H_B^2}$$

$$= \sqrt{4^2 + 12.22^2}\ \text{KN}$$

$$= 12.85\ \text{KN}$$

$$\tan\theta = \frac{V_A}{H_A} = \frac{4}{12.22}$$

$$\theta = 18°6'$$



Elements of Suspension Bridge

EQUILIBRIUM OF LIGHT CABLE: GENERAL CABLE THEOREM

Figure shows a light cord or cable suspended from two points *A* and *B* and subjected to a number of point loads $W_1, W_2 \ldots\ldots W_n$. Let *L* be the horizontal span of the cable and α be the inclination of the line *AB,* with the horizontal. Evidently, the difference in elevation between the two supports *A* and *B* is equal to *L* tan α.

Let V_A and V_B be the vertical components of reactions at *A* and *B.* Since there is no horizontal loading on the cable, the horizontal reaction (H) at the ends *A* and *B* will be equal in magnitude but opposite in direction. Since the cable is in equilibrium, it will take the shape of a funicular polygon for the load system and will therefore, deform as shown.

In order to find the vertical reaction V_A, take moments about B:

$$Hy = \frac{x}{L}\ \Sigma M_B - \ \Sigma M_x$$

This equation is known as general cable theorem.

UNIFORMLY LOADED CABLE

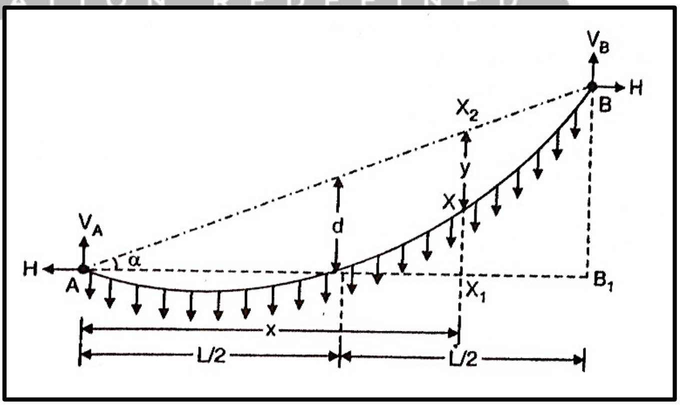
Expression for Horizontal Reaction

Figure shows a cable supporting a uniformly distributed load of intensity *p* per unit length. From the general cable theorem derived in the previous article we have

$$Hy = \frac{x}{L}\ \Sigma M_B - \ \Sigma M_x$$

Where,

$y = XX_2$ = vertical ordinate between the line *AB* and chord at the point *X*



$\Sigma M_B = pL \cdot \frac{L}{2} = p\frac{L^2}{2}$

$\Sigma M_x = px \cdot \frac{x}{2} = p\frac{x^2}{2}$

$Hy = \frac{x}{L} \cdot p\frac{L^2}{2} - p\frac{x^2}{2}$

$= p\frac{Lx}{2} - p\frac{x^2}{2}$

At the mid – span,

$x = L/2$ and $y = d = dip$ of the cable.

$$\therefore \qquad Hd = p\,\frac{L}{2}\frac{L}{2} - \frac{p}{2}\left(\frac{L}{2}\right)^2$$

$$= p\frac{L^2}{8}$$

Hence $H = p\frac{L^2}{8d}$ ……………. (2)

EXPRESSION FOR CABLE TENSION AT THE ENDS

The cable tension T at any end is the resultant of vertical and horizontal reaction at the end. Thus

$$T_A = \sqrt{{V_A}^2 + H^2} \text{ and } T_B = \sqrt{{V_B}^2 + H^2}$$

Knowing H from Eq. (2) And V_A from Equation (1) the cable tension T_A or T_B can be easily calculated. When the cable chord is horizontal, $V_A = V_B = \frac{pL}{2}$

Hence,

$$T_A = T_B = T = \sqrt{(\frac{PL}{2})^2 + (\frac{PL^2}{8d})^2}$$

$$T = \frac{pL}{2}\sqrt{1 + \frac{L^2}{16_{d^2}}}$$

$$T = H\sqrt{1 + \frac{16d^2}{L^2}}$$

The inclination β of T with the vertical is given by

$$\tan\beta = \frac{H}{V} = \frac{PL^2}{8d} \cdot \frac{2}{pL} = \frac{L}{4d}$$

It should be remembered that the horizontal component of cable tension at any point will be equal to H.

SHAPE OF THE CABLE

Let us now determine the shape of the cable under the uniformly distributed load. Substituting the value of H (Eq. 2) in Eq.

$$\text{Hy} = \frac{pLx}{2} - \frac{Px^2}{2}$$

$$\left(\frac{PL^2}{8d}\right) y = \frac{pLx}{2} - \frac{Px^2}{2}$$

EXAM ACHARYA
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GPSC - CIVIL

Transportation Engineering

END is not the end if fact E.N.D. means
" Effort Never dies"

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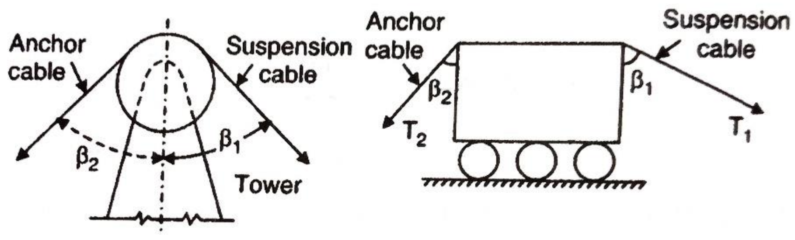
ANCHOR CABLES

The suspension cable is supported on either sides, on the supporting towers. The anchor cable transfer the tension of the suspension cable to the anchorage consisting of huge mass of concrete. There are generally two arrangements for this. The suspension cable can either be passed over the guide pulley for anchoring it to the other side or it can be attached to a saddle mounted on roller. Figure (*a*) and (*b*) show both the arrangements.



(a) Guide pulley support (b) Roller support

In the former case, when the suspension cable passes over the guide pulley and forms the part of the anchor cable to the other side, the tension *T* in the cable is the same on both the sides.

Let β_1 = inclination of the suspension cable, with vertical.

β_2 = inclination of the anchor cable, with vertical.

∴ Pressure on the top of pier = $V_p = T\cos\beta_1 + T\cos\beta_2$

$$= T(\cos\beta_1 + \cos\beta_2)$$

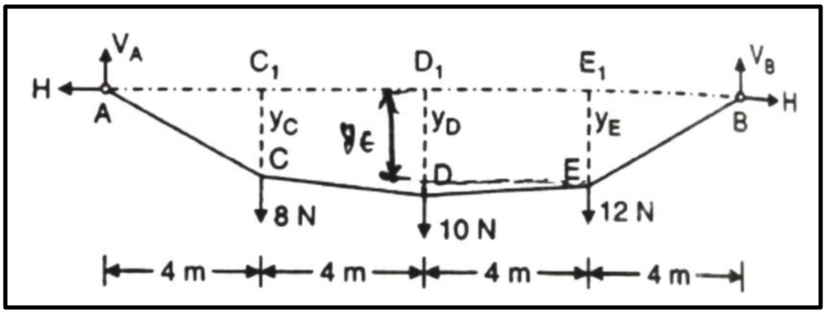
Horizontal force on the top of the pier

$$= T\sin\beta_1 - T\sin\beta_2 = T(\sin\beta_1 - \sin\beta_2)$$

This horizontal force will cause bending moment in the tower.

If the cable is supported on a saddle mounted on rollers, as shown in Figure (b), The horizontal components of the tensions in the suspension cable and the anchor cable will be equal since the rollers do not have any horizontal reaction.

$$\therefore T_1\sin\beta_1 = T_2\sin\beta_2 = H$$



$$AC = \sqrt{16 + Y_C^2}$$

$$= 4\sqrt{1 + 0.0625Y_C^2}$$

$$CD = \sqrt{16 + (Y_D - Y_C)^2}$$

$$CD = \sqrt{16 + (\frac{10}{7} - 1)^2 Y_C^2}$$

$$= \sqrt{16 + \frac{9}{49} Y_C^2}$$

$$= 4\sqrt{1 + 0.0115Y_C^2}$$

$$DE = \sqrt{16 + (Y_D - Y_E)^2}$$

$$DE = \sqrt{16 + (\frac{10}{7} - \frac{8}{7})^2 Y_C^2} = \sqrt{16 + \frac{4}{49} Y_C^2} = 4\sqrt{1 + 0.0051Y_C^2}$$

$$EB = \sqrt{16 + {y_E}^2} = \sqrt{16 + (\frac{8}{7} Y_C)^2} = \sqrt{16 + \frac{64}{49} Y_C^2} = 4\sqrt{1 + 0.0816Y_C^2}$$

Total length $AB = AC + CD + DE + EB$

$18 = 4\,[(1 + 0.0625\,Y_C^2)^{1/2} + (1 + 0.0115Y_C^2)^{1/2} + (1 + 0.0051Y_C^2)^{1/2} + (1 + 0.0816Y_C^2)^{1/2}]$

$4.5 \approx [\,(1 + \frac{0.0625}{2}\,Y_C^2\,) + (\,1 + \frac{0.0115}{2}\,Y_C^2) + (\,1 + \frac{0.0051}{2}\,Y_C^2) + (\,1 \frac{0.0816}{2}\,Y_C^2)\,]$

$4.5 = [4 + 0.08\,Y_C^2]$

$Y_C = 2.5$ m ; $Y_D = \frac{10}{7}\,Yc = 3.57\,m$ and $Y_E = \frac{8}{7}\,Yc = 2.86\,m$

Thus, with the known values of Y_C, Y_D and $Y_{E,}$ the shape of the cable is determined.

In order to find the horizontal reaction H, apply the general cable theorem at point C.

$$v_A = \frac{1}{2}(5+5+5+5+5) = 12.5 \text{ kN}$$

$$\therefore CC_1 = GG_1 = \mu_C = \text{n}\ (12.5 \text{ x } 5) = 62.5$$

$$DD_1 = FF_1 = \mu_D = (12.5 \text{ x } 10) - (5 \text{ x } 5) = 100$$

$$EE_1 = \mu_E = (12.5 \text{ x } 15) - (5 \text{ x } 10) - (5 \text{ x } 5) = 112.5$$

$$\therefore EE_1 : DD_1 : CC_1 : : 112.5 : 100 : 62.5 \text{ or } Y_E : Y_D : Y_C : : 1 : 0.89 : 0.556$$

$$Y_E = 2.5 \text{ m}$$

$$\therefore Y_D = Y_F = 2.5 \text{ x } 0.89 = 2.22 \text{ m and } Y_C = Y_G = 2.5 \text{ x } 0.556 = 1.39 \text{ m}$$

The length of the cable = 2 (*AC* + *CD* +*DE)*

$$= 2\ [5\ \{1 + \frac{1.39^2}{25}\}^{\frac{1}{2}} + 5\{1 + \frac{(2.22-1.39)^2}{25}\}^{\frac{1}{2}} + 5\ \{1 + \frac{(2.5-2.22)^2}{25}\}^{\frac{1}{2}}]$$

$$= 10\ [1+\frac{19.3}{50} + 1 + \frac{0.69}{50} + 1 + \frac{0.08}{50}] = 30.54 \text{ m}$$

The length of the cable can also be found approximately by treating the string as a parabola. In that case,

$$s = L + \frac{8}{3}\frac{d^2}{L} = 30 + \frac{8}{3}\frac{(2.5)^2}{30} = 30.56 \text{ m}$$

To find the horizontal reaction *H*, take moment about *C* of all forces to the left of it and equate it to zero. Thus,

$$M_c = 0 = (H \text{ x } 1.39) - V_A \text{ x } 5 = 1.39H - 5 \text{ x } 12.5$$

$$\therefore H = \frac{5 \text{ x } 1.25}{1.39} = 45 \text{ kN}$$

The maximum tension in *AC* = $\sqrt{(45)^2 + (12.5)^2}$ = 46.6 kN

$$\therefore \text{Area required} = \frac{46.6 \text{ x } 1000}{140} = 333\ mm^2$$

Example: A flexible rope weighing I N per metre span between two points 40 m apart and at the same level, 12 m above the ground. It is to carry a concentrated load of 300 N at a point P on the rope which is to be at a horizontal distance of 10 m from the left hand support. What is the maximum height above the ground to which the point P may

CHAPTER – 9

INFLUENCE LINE DIAGRAM

Influence line diagram is the graphical representation of effects of a moving load on a span. By ILD variations of support reactions, shear force and bending moment at any section of the span are obtained due to a rolling load from one end to another.

MULLER-BRESLAU PRINCIPAL

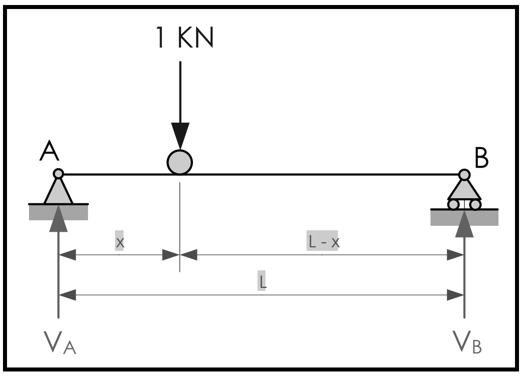
The Muller-Breslau influence theorem for statically determinate beams may be stated as follows,

"The influence line for an assigned function of a statically determinate beam may be obtained by removing the restraint offered by that function and introducing a directly related generalized unit displacement at the location and in the direction of the function."

Case 1: ILD for Support Reaction

Consider a simply supported beam AB of length L. A unit load is moving from left to right support.

∴ The support reactions are,



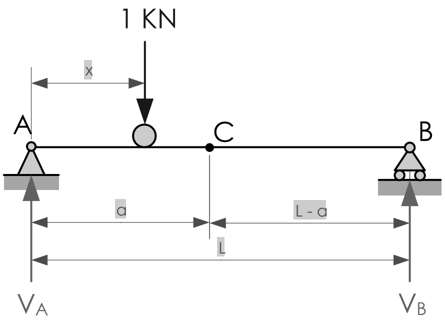
Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
Total full length test	: 13
Mock test	: 16
Total test	: 80

Conclusion

To draw the ILD for reaction at support lift the beam by unit amount at that support and the corresponding deflected shape is the ILD that support.

Case 2: ILD for Shear Force



Moment about A,

$$\sum M_A = 0$$

$$x - LV_B = 0$$

$$V_B = \frac{x}{L} KN$$

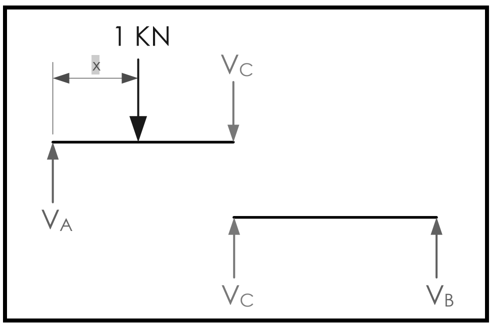
Taking all the vertical forces,

$$\sum V = 0$$

$$V_A + V_B - 1 = 0$$

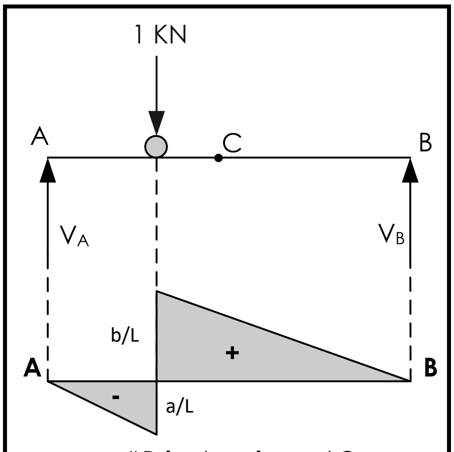
$$V_A = \frac{L-x}{L} KN$$

When unit load is in AC span,



So, at x = a + b,

$$V_C = 0 \text{ KN}$$

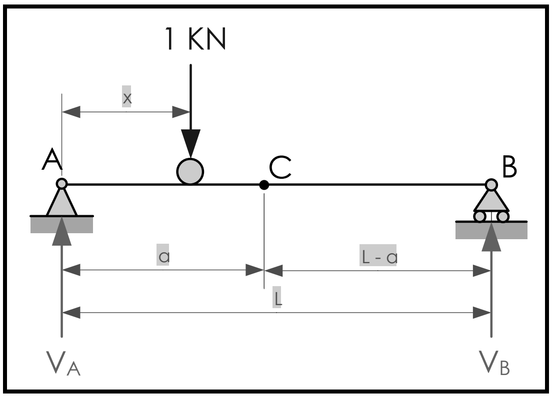


ILD for shear force at C

Conclusion

To draw ILD for S.F. at any section then cut the beam at that section and displace the both part in opposite direction as shown in figure in the ratio of their respective length to the length of span.

Case 3: ILD for Bending Moment



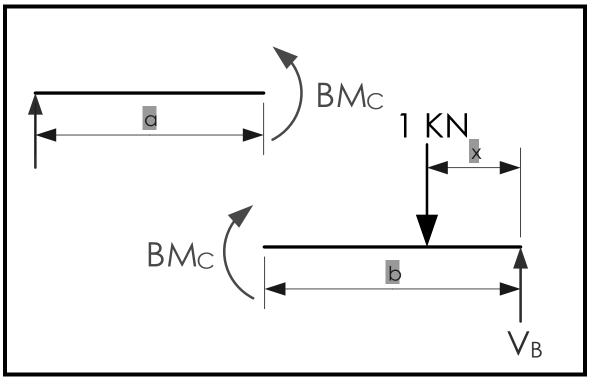
Moment about A,

$$\sum M_A = 0$$

$$x - LV_B = 0$$

$$V_B = \frac{x}{L} \text{ KN}$$

When unit load is in CB span,



$$BM_C = aV_A$$

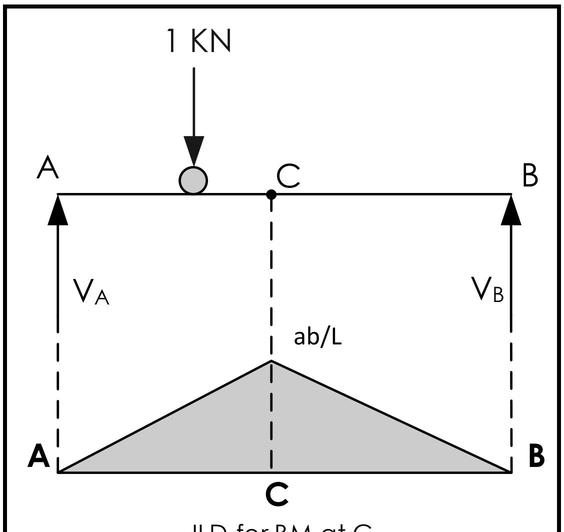
$$= \frac{a(L-x)}{L}$$

At, x = a,

$$BM_C = \frac{ab}{L}$$

At, x = L,

$$BM_C = 0 \text{ KN}$$



ILD for BM at C

GPSC - CIVIL

Water Resource Engineering

EXAM ACHARYA
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"Don't Fear for Facing Failure in the First Attempt, Because even the Successful Maths Start with 'Zero' only."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

By using similar triangle property, we can calculate the values of x, y and z.

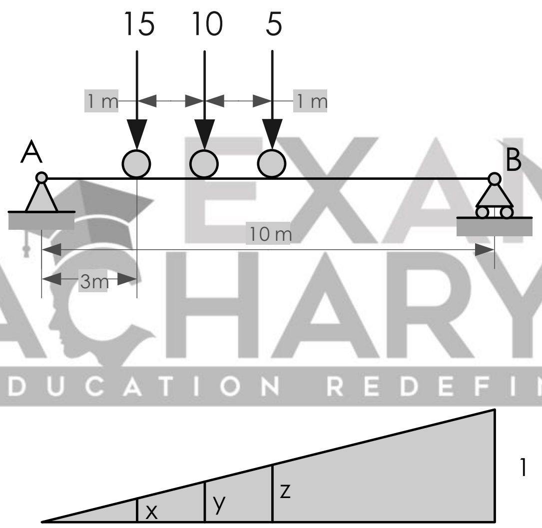
$$\frac{1}{10} = \frac{x}{7} \rightarrow x = 0.7 \text{ m}$$

$$\frac{1}{10} = \frac{y}{6} \rightarrow y = 0.6 \text{ m}$$

$$\frac{1}{10} = \frac{z}{5} \rightarrow z = 0.5 \text{ m}$$

$$\therefore R_A = (15 \times 0.7) + (10 \times 0.6) + (5 \times 0.5) = 19 \text{ KN}$$

ILD for RB,



ILD for R_B

By using similar triangle property, we can calculate the values of x, y and z.

$$\frac{1}{10} = \frac{x}{3} \rightarrow x = 0.3 \text{ m}$$

$$\frac{1}{10} = \frac{y}{4} \rightarrow y = 0.4 \text{ m}$$

$$\frac{1}{10} = \frac{z}{5} \rightarrow z = 0.5 \text{ m}$$

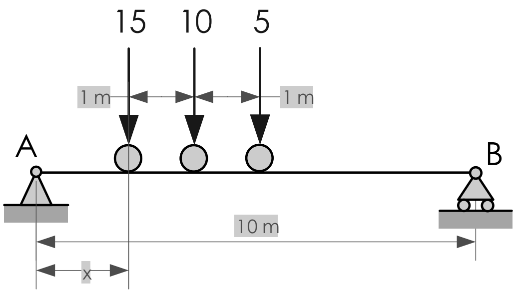
$$\therefore R_A = (15 \times 0.3) + (10 \times 0.4) + (5 \times 0.5) = 11 \text{ KN}$$

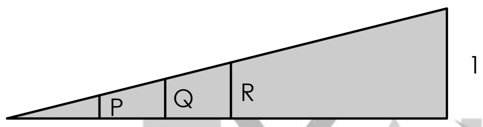
Answer

The reaction at A and B when series is on a distance 3m from left support is 19 KN and 11 KN respectively.

$$\therefore R_A = \left(15 \times \frac{100-x}{10}\right) + \left[10 \times \frac{10-(x+1)}{10}\right] + \left[5 \times \frac{10-(x+2)}{10}\right] \text{ KN}$$

ILD for R_B,





ILD for R_B

By using similar triangle property, we can calculate the values of P, Q and R.

$$\frac{1}{10} = \frac{P}{x} \rightarrow P = \frac{x}{10} \text{ m}$$

$$\frac{1}{10} = \frac{Q}{x+1} \rightarrow Q = \frac{x+1}{10} \text{ m}$$

$$\frac{1}{10} = \frac{R}{x+2} \rightarrow R = \frac{x+2}{10} \text{ m}$$

$$\therefore R_B = \left(15 \times \frac{x}{10}\right) + \left(10 \times \frac{x+1}{10}\right) + \left(5 \times \frac{x+2}{10}\right) \text{ KN}$$

According to the question,

$$R_A = R_B$$

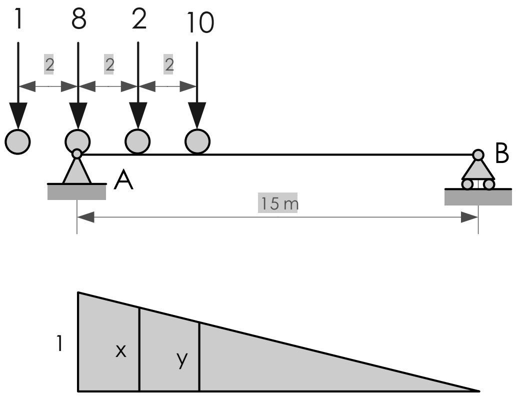
$$\left(15 \times \frac{100-x}{10}\right) + \left[10 \times \frac{10-(x+1)}{10}\right] + \left[5 \times \frac{10-(x+2)}{10}\right]$$
$$= \left(15 \times \frac{x}{10}\right) + \left(10 \times \frac{x+1}{10}\right) + \left(5 \times \frac{x+2}{10}\right)$$

$$x = 4.33 \text{ m}$$

Answer

At 4.33 m from the left support R_A and R_B are same.

Case – 2: when 8KN is at A



ILD for R_A

By using similar triangle property, we can calculate the values of x, y and z.

$$\frac{1}{15} = \frac{x}{13} \rightarrow x = \frac{13}{15}$$

$$\frac{1}{15} = \frac{y}{11} \rightarrow y = \frac{11}{15}$$

$$\therefore R_A = (8 \times 1) + \left(2 \times \frac{13}{15}\right) + \left(10 \times \frac{11}{15}\right) \text{KN}$$

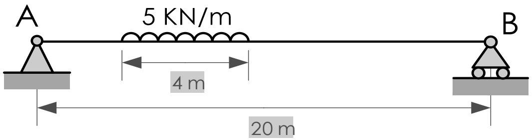
$$= 17.06\text{KN} > 15.4\text{KN}$$

∴ The maximum reaction at A is 17.06 KN.

Answer

The maximum reaction at A is 17.06 KN.

Q4. A ULD of intensity 5KN/m and length 4m is moving on a span of 20 m from left support to right support. What will be the location of UDL when reaction at left support is twice of reaction at support B.



Solution

Let at a distance of ‘x’ the reaction at A is twice of reaction at B,

$$\therefore R_B = 5 \times \text{shaded area (area of trapezium)}$$

$$= 5 \times \frac{1}{2} \times 4 \times \left[\frac{x}{20} + \frac{x+4}{20}\right]$$

$$= x + 2$$

According to the question,

$$R_A = 2R_B$$

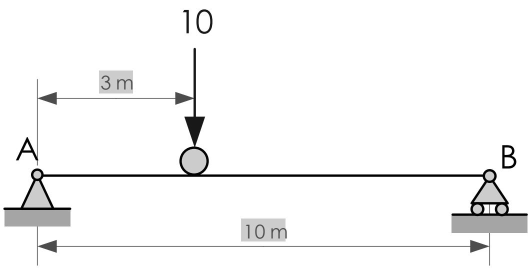
$$18 - x = 2(x + 2)$$

$$x = \frac{14}{3}\text{ m} = 4.67\text{ m}$$

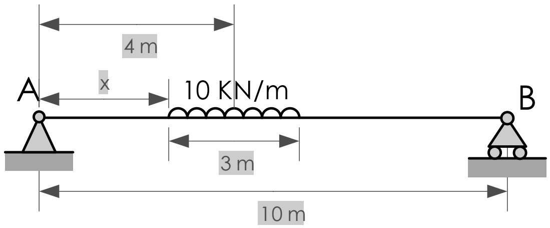
Answer

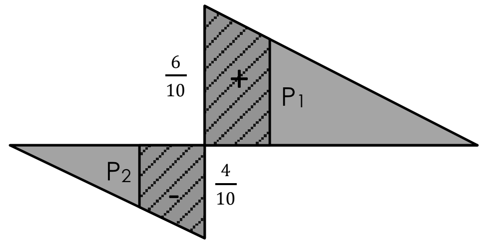
The distance from the left support where the reaction at A is twice of reaction at B is 4.67 m.

Q5. A load of 10KN is moving from left to right support. Draw the ILD for S.F. on a section 3m from left support and also determine the maximum positive S.F. on this section (length of span = 10 m).



Solution





By using similar triangle property, we can calculate the values of P1 and P2.

$$\frac{\frac{6}{10}}{6} = \frac{P_1}{10-(x+3)} \rightarrow P_1 = \frac{10-(x+3)}{10}$$

$$\frac{\frac{4}{10}}{4} = \frac{P_2}{x} \rightarrow P_2 = \frac{x}{10}$$

$$\therefore SF = 10 \times \left\{\left[\frac{1}{2} \times \left(\frac{10-(x+3)}{10} + \frac{4}{10}\right) \times (4-x)\right]\right.$$

$$\left. - \left[\frac{1}{2} \times \left(\frac{x}{10} + \frac{4}{10}\right) \times (x+3-4)\right]\right\}$$

$$= 145 - 70x$$

According to the questions,

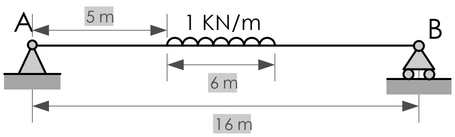
$$SF = 0$$

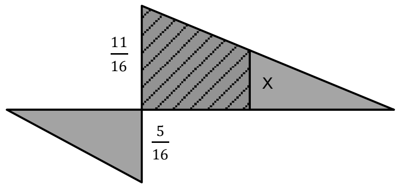
$$\therefore 145 - 70x = 0$$

$$x = 2.07 \text{ m}$$

Answer:

The location of load when net S.F. at a section 4m from left support is zero is 2.07 m from left support.





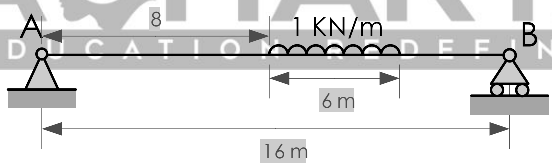
By using similar triangle property, we can calculate the values of x,

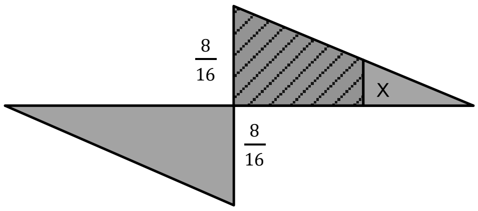
$$\frac{\frac{11}{16}}{13} = \frac{x}{5} \rightarrow x = \frac{5}{16}$$

$$\therefore (SF)_{max} = 1 \times \left[\frac{1}{2} \times 6 \times \left(\frac{11}{16} + \frac{5}{16}\right)\right] \text{ KN}$$

$$= 3 \text{ KN}$$

Maximum shear force at 8 m from left support,





By using similar triangle property, we can calculate the values of x,

$$\frac{\frac{8}{16}}{8} = \frac{x}{2} \rightarrow x = \frac{2}{16}$$

$$\therefore (SF)_{max} = 1 \times \left[\frac{1}{2} \times 6 \times \left(\frac{8}{16} + \frac{2}{16}\right)\right] \text{ KN}$$

$$= 1.875 \text{ KN}$$

Test Series Available..

Total weekly test	***: 35***
Total mid subject test	***: 16***
Total full length test	***: 13***
Mock test	***: 16***
Total test	***: 80***

When 8 KN load is above the point C,

$$\mathrm{SF}_1 = \left(5 \times \frac{13}{20}\right) - \left(8 \times \frac{5}{20}\right) - \left(6 \times \frac{3}{20}\right) - \left(4 \times \frac{8}{20}\right) \text{ KN}$$

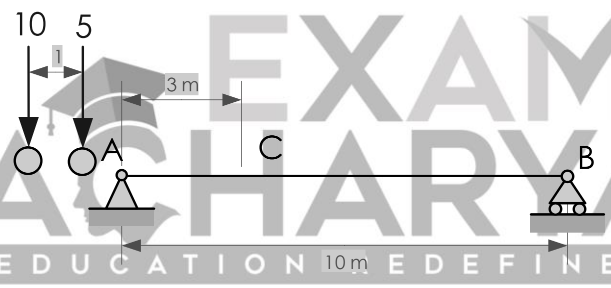
$$= 0.15 \text{ KN (+ve)}$$

So, the maximum negative shear is - 2.75 KN.

Answer:

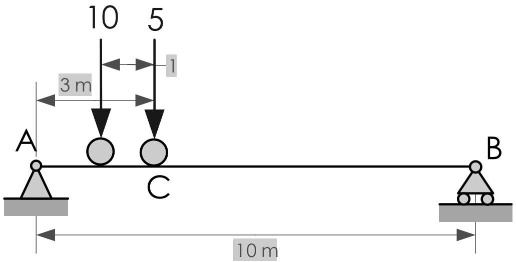
The maximum negative shear is - 2.75KN.

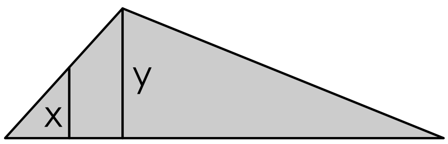
Q9. Determine the maximum BM at a section C, distance by 3 m from support A, due to movement of 2 wheel loads 5KN and 10KN, 1m apart from each other on a span of 10m.



Solution

Case – 1:

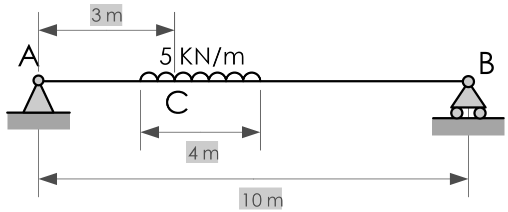




We know that,

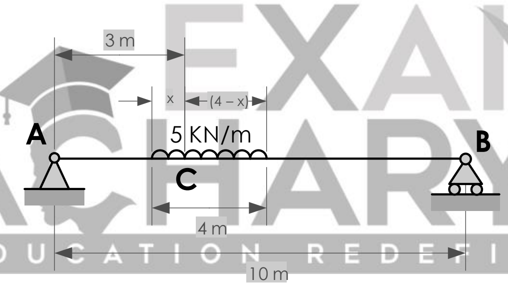
$$y = \frac{ab}{L} = \frac{3 \times 7}{10} \text{ m} = 2.1 \text{ m}$$

Q10. What will be the maximum BM on C if a UDL of intensity 5KN/m and length of 4m runs from left to right then what will be the maximum BM on C.



Solution

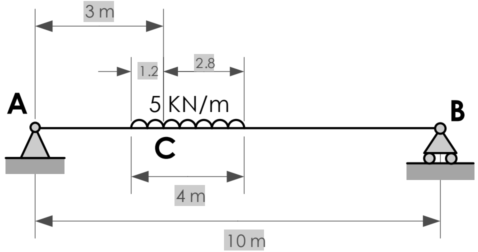
Let us suppose that loading is placed over section C in such a way that its ‘x’ length is in the side of CA and remaining (4 – x) length is in the side of CB portion.

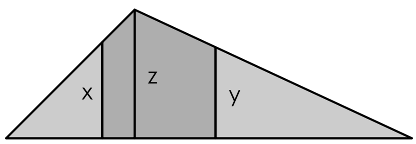


To develop the maximum BM on C the loading should be divided on both sides of C in the ratio of span so that average on both sides of C should be same.

$$\text{i.e. } \frac{5x}{3} = \frac{5(4-x)}{7}$$

$$\text{or, } x = 1.2 \text{ m}$$





When 4 KN load is above C,

$$BM = (4 \times 4.55) + (10 \times 3.85) + (13 \times 3.15) + (2 \times 2.45) \text{ KN-m}$$

$$= 102.55 \text{ KN-m}$$

When 10 KN load is above C,

$$BM = (4 \times 3.25) + (10 \times 4.55) + (13 \times 3.85) + (2 \times 3.15) \text{ KN-m}$$

$$= 114.85 \text{ KN-m}$$

When 13 KN load is above C,

$$BM = (4 \times 1.95) + (10 \times 3.25) + (13 \times 4.55) + (2 \times 3.85) \text{ KN-m}$$

$$= 107.15 \text{KN-m}$$

So, the maximum bending moment is 114.85 KN-m

Answer

The maximum bending moment is 114.85 KN-m